Neutrino Oscillations (12)

$$\begin{pmatrix} v_e \\ v_\mu \\ v_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

 $v_e \rightarrow v_{\mu\tau}$ Oscillations Solar neutrinos & Reactor neutrinos

1



Energy production in stars

Hans Bethe 1939 (Nobel prize 1967)

MARCH 1, 1939

PHYSICAL REVIEW

VOLUME 55

Energy Production in Stars*

H. A. BETHE Cornell University, Ithaca, New York (Received September 7, 1938)

It is shown that the most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons. These reactions form a cycle in which the original nucleus is reproduced, viz. $C^{12}+H=N^{13}$, $N^{13}=C^{12}+e^+$, $C^{12}+H=N^{14}$, $N^{14}+H=O^{13}$, $O^{12}=N^{13}+e^+$, $N^{13}+H=C^{12}$ $+He^4$. Thus carbon and nitrogen merely serve as catalysts for the combination of four protons (and two electrons) into an *c*-particle (§7).

The carbon-nitrogen reactions are unique in their cyclicial character (\$8). For all nuclei lighter than carbon, reaction with protons will lead to the emission of an α -particle so that the original nucleus is permanently destroyed. For all nuclei heavier than fluorine, only radiative capture of the protons occurs, also destroying the original nucleus. Oxygen and fluorine reactions mostly lead back to nitrogen. Besides, these heavier nuclei react much more slowly than C and N and are therefore unimportant for the energy production.

The agreement of the carbon-nitrogen reactions with observational data (§7, 9) is excellent. In order to give the correct energy evolution in the sun, the central temperature of the sun would have to be 18.5 million degrees while

§1. INTRODUCTION

THE progress of nuclear physics in the last few years makes it possible to decide rather definitely which processes can and which cannot occur in the interior of stars. Such decisions will be attempted in the present paper, the discussion being restricted primarily to main sequence stars. The results will be at variance with some current hypotheses.

The first main result is that, under present conditions, no elements heavier than helium can be built up to any appreciable extent. Therefore we must assume that the heavier elements were built up *before* the stars reached their present state of temperature and density. No attempt will be made at speculations about this previous state of stellar matter.

The energy production of stars is then due entirely to the combination of four protons and two electrons into an *α*-particle. This simplifies the discussion of stellar evolution inasmuch as

* Awarded an A. Cressy Morrison Prize in 1938, by the New York Academy of Sciences.

integration of the Eddington equations gives 19. For the brilliant star Y Cygni the corresponding figures are 30 and 32. This good agreement holds for all bright stars of the main sequence, but, of course, not for giants,

For fainter stars, with lower central temperatures, the reaction $H+H=D+e^{+}$ and the reactions following it, are believed to be mainly responsible for the energy production. (§10)

It is shown further (§5-6) that no elements heavier than He⁴ can be built up in ordinary stars. This is due to the fact, mentioned above, that all elements up to boron are disintegrated by proton bombardment (α -emission!) rather than built up (by radiative capture). The instability of Be⁵ reduces the formation of heavier elements still further. The production of neutrons in stars is likewise negligible. The heavier elements found in stars must therefore have existed already when the star was formed.

Finally, the suggested mechanism of energy production is used to draw conclusions about astrophysical problems, such as the mass-luminosity relation (§10), the stability against temperature changes (§11), and stellar evolution (§12).

the amount of heavy matter, and therefore the

The combination of four protons and tw electrons can occur essentially only in two ways The first mechanism starts with the combinatio of two protons to form a deuteron with positro emission, viz.

 $H+H=D+\epsilon^+$.

The deuteron is then transformed into He⁴ b further capture of protons; these captures occu very rapidly compared with process (1). Th second mechanism uses carbon and nitrogen a catalysts, according to the chain reaction

$C^{12} + H = N^{13} + \gamma$,	$N^{13} = C^{13} + \epsilon^+$
$C^{13} + H = N^{14} + \gamma$,	
$N^{14} + H = O^{15} + \gamma$,	$O^{15} = N^{15} + \epsilon^+$
$N^{15} + H = C^{12} + He^4$.	

The catalyst C¹² is reproduced in all cases except about one in 10,000, therefore the abundance of carbon and nitrogen remains practically unchanged (in comparison with the change of the number of protons). The two reactions (1) and 434



The combination of four protons and two electrons can occur essentially only in two ways. <u>The first mechanism</u> starts with the combination of two protons to form a deuteron with positron emission, *viz*.

$$\mathbf{H} + \mathbf{H} = \mathbf{D} + \boldsymbol{\epsilon}^+. \tag{1}$$

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$$N^{14} + H = O^{15} + \gamma, \qquad O^{15} = N^{15} + \epsilon^{+}$$

$$N^{15} + H = C^{12} + He^{4}.$$
(2)

Energy production in the sun: pp cycle



Solar Neutrino Spectrum



Calculation and Measurement of the flux of solar neutrinos

Bahcall, Davis 1964

VOLUME 12, NUMBER 11

PHYSICAL REVIEW LETTERS

16 March 1964

SOLAR NEUTRINOS. I. THEORETICAL*

California Institute of Technology, Pasadena, California (Received 6 January 1964)

The principal energy source for main-sequence stars like the sun is believed to be the fusion, in the deep interior of the star, of four protons to form an alpha particle.¹ The fusion reactions are thought to be initiated by the sequence ${}^{1}\mathrm{H}(\rho, \gamma){}^{3}\mathrm{H}e$ and terminated by the following sequences: (i) ${}^{3}\mathrm{He}({}^{3}\mathrm{He}, 2\rho){}^{4}\mathrm{He}$; (ii) ${}^{3}\mathrm{He}(\alpha, \gamma){}^{7}\mathrm{Be}(e^{-}\nu){}^{7}\mathrm{Li}(\rho, \alpha){}^{4}\mathrm{He}$; and (iii) ${}^{3}\mathrm{He}(\alpha, \gamma){}^{7}\mathrm{Be}(\rho, \gamma){}^{9}\mathrm{B}$. $(e^{+}\nu){}^{9}\mathrm{Be}{}^{*}(\alpha){}^{4}\mathrm{He}$. No <u>direct</u> evidence for the existence of nuclear reactions in the interiors of stars has yet been obtained because the mean free path for photons emitted in the center of a star is typically less than 10^{-10} of the radius of the star. Only neutrinos, with their extremely small interaction cross sections, can enable us to see into the interior of a star and thus verify directly the hypothesis of nuclear energy generation in stars.

The most promising method² for detecting solar neutrinos is based upon the endothermic reaction $(Q = -0.81 \text{ MeV}) \, {}^{37}\text{Cl}(\nu_{\text{solar}}, e^{-}) {}^{37}\text{Ar}$, which was first discussed as a possible means of detecting neutrinos by Pontecorvo³ and Alvarez.⁴ In this note, we predic the number of absorptions of

SOLAR NEUTRINOS. II. EXPERIMENTAL

Raymond Davis, Jr. Chemistry Department, Brookhaven National Laboratory, Upton, New Yo (Received 6 January 1964)

The prospect of observing solar neutrinos by means of the inverse beta process ${}^{37}\text{Cl}(\nu, e^{-}){}^{37}\text{Ar}$ induced us to place the apparatus previously described¹ in a mine and make a preliminary search. This experiment served to place an upper limit on the flux of extraterrestrial neutrinos. These results will be reported, and a discussion will be given of the possibility of extending the sensitivity of the method to a degree capable of measuring the solar neutrino flux calculated by Bahcall in the preceding paper.²

The apparatus consists of two 500-gallon tanks of perchlorethylene, C_gCl_4 , equipped with agitators and an auxiliary system for purging with helium. It is located in a limestone mine 2300 feet below the surface³ (1800 meters of water equivalent shielding, m.w.e.). Initially the tanks were swept completely free of air argon by purging the tanks with a stream of helium gas. ³⁶Ar carrier (0.10 cm³) was introduced and the tanks exposed for periods of four months or more to allow the 35-d ³⁷Ar activity to reach nearly the saturation value. Carrier argon along with any ³⁷Ar pro3 counts in 18 days is probably entirely due to the background activity. However, if one assumes that this rate corresponds to real events and uses the efficiencies mentioned, the upper limit of the neutrino capture rate in 1000 gallons of C_2Cl_4 is ≤ 0.5 per day or $\varphi \overline{\sigma} \leq 3 \times 10^{-34} \text{ sec}^{-1} ({}^{37}\text{Cl atom})^{-1}$. From this value, Bahcall² has set an upper limit on the central temperature of the sun and other relevant information.

On the other hand, if one wants to measure the solar neutrino flux by this method one must use a much larger amount of C_2Cl_4 , so that the expected ³⁷Ar production rate is well above the background of the counter, 0.2 count per day. Using Dahcall's expression,

 $\sum \varphi_{v}(\text{solar}) \sigma_{\text{abs}}$

 $= (4 \pm 2) \times 10^{-35} \text{ sec}^{-1} ({}^{37}\text{Cl atom})^{-1},$

then the expected solar neutrino captures in 100000 gallons of $C_2 Cl_4$ will be 4 to 11 per day, which is an order of magnitude larger than the counter background. On the basis of experience

R. Davis & J. Bahcal

the star. Only neutrinos, with their extremely small interaction cross sections, can enable us to see into the interior of a star and thus verify directly the hypothesis of nuclear energy generation in stars.



GALLEX (& GNO)



$$v_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^- \qquad E_v > 233 \text{ keV}$$



Ge – Counting





active side

GALLEX



$$e^{-}+{}^{71}\text{Ge} \rightarrow {}^{71}\text{Ga} + \text{v}_{e}$$

 $T_{1/2} = 11.4$
d

Gallex / GNO results



*) 1 SNU (solar neutrino unit) = 1 v-capture / 10³⁶ target atoms

The solar neutrino puzzle

Total Rates: Standard Model vs. Experiment Bahcall-Serenelli 2005 [BS05(0P)]





Creighton Mine (Nickel) Sudbury, Canada

Depth 2070m

1000t D₂O

AND AND AND

9500 PMTs

WH TELLEN



Neutrino detection in

$$\begin{array}{ccc} \mathbf{CC} & v_e + d \rightarrow p + p + e^- \\ \mathbf{ES} & v_e + e^- \rightarrow v_e + e^- \\ \mathbf{NC} & v_x + d \rightarrow p + n + v_x \end{array}$$







SNO results

"Independent Measurement of the Total Active 8B Solar Neutrino Flux Using an Array of 3He Proportional Counters at the Sudbury Neutrino Observatory", SNO Collaboration, PRL 101, 111301 (2008):

CC
$$v_e + d \rightarrow p + p + e^-$$

ES $v_e + e^- \rightarrow v_e + e^-$
NC $v_x + d \rightarrow p + n + v_x$

$$\phi_{\rm CC}^{\rm SNO} = 1.67^{+0.05}_{-0.04}(\text{stat})^{+0.07}_{-0.08}(\text{syst})$$

$$\phi_{\rm ES}^{\rm SNO} = 1.77^{+0.24}_{-0.21}(\text{stat})^{+0.09}_{-0.10}(\text{syst})$$

$$\phi_{\rm NC}^{\rm SNO} = 5.54^{+0.33}_{-0.31}(\text{stat})^{+0.36}_{-0.34}(\text{syst}),$$

(in units of $10^6 \text{ cm}^{-2} \text{ s}^{-1}$)

$$\frac{\phi_{\rm CC}^{\rm SNO}}{\phi_{\rm NC}^{\rm SNO}} = 0.301 \pm 0.033 \text{(total)}$$

- 1/3 of solar neutrinos are detected as v_e in SNO.
- 2/3 of solar neutrinos are detected as v_{μ} or v_{τ} in SNO.
- measured total neutrino flux = SSM predicted flux

Neutrino Oscillation Analysis (after SNO)

Global Analysis using data from SNO, SK, Cl, Ga, Borexino



SNO coll., PRL 101, 111301 (2008)

Solution of the solar neutrino puzzle



Explanation of results of solar neutrino experiments: (\approx 60% of pp-neutrinos and \approx 33% of ⁸B neutrinos arrive on earth as v_e)

- We assume: $\Delta m_{12}^{\ 2} = 8 \bullet 10^{-5} eV^2, \ \theta_{12} \approx 33^{\circ}$
- The probability of vacuum oscillations is then given by:

$$P(v_e \to v_e) = 1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{1.27 \frac{\Delta m_{12}^2 / \text{eV}^2 \cdot L/\text{m}}{E / \text{MeV}}}{\text{MeV}} \right)$$

$$\delta = 1.27 \frac{8 \cdot 10^{-5} \cdot 1.5 \cdot 10^{11}}{\text{values of } 0.1 - 10} = 10^{7\pm 1}$$

The phase varies over a large range!

• Therefore the survival probability for solar neutrinos (in vacuum) is:

$$\langle P(v_e \rightarrow v_e) \rangle_{\text{averaging over phases}} = 1 - \frac{1}{2} \sin^2 2\theta_{12} \approx 0.6$$

This explains quite well the experimental results for pp-neutrinos

But this is independent of energy!

 \rightarrow Therefore we need another effect to explain why only 33% of ⁸B neutrinos arrive...

Lecture ²

Neutrino propagation in matter – MSW (Mikheyev, Smirnov, Wolfenstein) Effect



In matter there is an additional potential in the equation of motion for $ve \rightarrow ve$ scattering (Flavor base)

Matter:
$$i \frac{d}{dt} \begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta + 2\sqrt{2}G_F N_e E & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e E \end{pmatrix} \begin{pmatrix} v_e \\ v_\mu \end{pmatrix}$$

with $2\sqrt{2}G_F \frac{N_e}{\frac{Y_e \rho}{m}} E = 1.53 \cdot 10^{-7} \text{ eV}^2 \left(\frac{Y_e \rho}{\text{g/cm}^3} \cdot \frac{E}{\text{MeV}}\right)$ center of Sun: $\frac{Y_e \rho}{\text{g/cm}^3} \cong 100$

Caren Hagner, Universität Hamburg

$$i\frac{d}{dt} \begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta + 2\sqrt{2}G_F N_e E & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e E \end{pmatrix} \begin{pmatrix} v_e \\ v_\mu \end{pmatrix}$$

can be written as:

$$i\frac{d}{dt}\begin{pmatrix}v_e\\v_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m_m^2\cos 2\theta_m & \Delta m_m^2\sin 2\theta_m\\\Delta m_m^2\sin 2\theta_m & \Delta m_m^2\cos 2\theta_m\end{pmatrix}\begin{pmatrix}v_e\\v_\mu\end{pmatrix}$$

where $\Delta m_m^2 = m_2^2 - m_1^2$ and θ_m , denote the "effective" masses and mixing angles in matter and:

$$\Delta m_m^2 \cos 2\theta_m = \Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e E$$
$$\Delta m_m^2 \sin 2\theta_m = \Delta m^2 \sin 2\theta$$

Solving these equations gives:

$$\Delta m_m^2 = \sqrt{\left(\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e E\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$
$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(\frac{2\sqrt{2}G_F N_e E}{\Delta m^2} - \cos 2\theta\right)^2 + \left(\sin 2\theta\right)^2}}$$

Caren Hagner, Universität Hamburg

Neutrino propagation in matter: 3 regimes

 $\Delta m_m^2 \cong \Delta m^2, \qquad \theta_m \cong \theta$ $2\sqrt{2}G_F N_e E = \Delta m^2 \cos 2\theta$ 2. Resonance: (Mikheyev, Smirnov 1985) $\Delta m_m^2 = \Delta m^2 \sin^2 2\theta$ $\theta_m = \frac{\pi}{4} \quad (45^\circ)$ $2\sqrt{2}G_F N_e E >> \Delta m^2 \cos 2\theta$ ⁸B Neutrinos 3. Matter dominated: $\Delta m_m^2 \to 2\sqrt{2}G_F N_e E$ $\theta_m \rightarrow \frac{\pi}{2}$ (90°)

 $2\sqrt{2}G_F N_\rho E \ll \Delta m^2 \cos 2\theta$

1. Quasi – vacuum:

pp, ⁷Be Neutrinos

Neutrino mixing in matter:

$$\begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} v_{1m} \\ v_{2m} \end{pmatrix}$$

⁸B neutrinos at center of Sun: $\theta_m = 73^\circ,$ with: $\Delta m_{12}^2 = 8 \cdot 10^{-5} eV^2,$ $\theta_{12} \approx 33^\circ,$ $Y\rho \approx 90g/cm^3.$

and

$$\begin{pmatrix} v_{1m} \\ v_{2m} \end{pmatrix} = \begin{pmatrix} \cos \theta_m & -\sin \theta_m \\ \sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} v_e \\ v_\mu \end{pmatrix}$$

For ⁸B neutrinos at the center of Sun:

$$\begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = \begin{pmatrix} \cos 73^\circ & \sin 73^\circ \\ -\sin 73^\circ & \cos 73^\circ \end{pmatrix} \begin{pmatrix} v_{1m} \\ v_{2m} \end{pmatrix}$$

The probability that the created v_e is in the state v_{2m} is then given by $sin^2(73^\circ) = 91\%!$

On its way through the Sun, the neutrino is with a constant probability of 91% in the state v_{2m} . But the composition of v_{2m} from v_e and v_u changes with the varying density.

Lecture 1

Life of a Boron-8 Solar Neutrino:



From Stephen Parke <u>http://boudin.fnal.gov/AcLec/AcLecParke.html</u> (par

Explanation of ⁸B solar neutrino results with matter ^{Lecture} effect





the detector observes the amount of $v_e in v_2$

The probability that a ⁸B neutrino is detected as v_e is then given by:

$$\langle P(v_e \rightarrow v_e) \rangle_{\text{incoherent}} = f_1 \cos^2 \theta + f_2 \sin^2 \theta$$

where:

 f_1 = probability that a neutrino leaving the Sun is in the state $v_1 = \cos^2(73^\circ) = 0.09$.

 f_2 = probability that a neutrino leaving the Sun is in the state $v_2 = \sin^2(73^\circ) = 0.91$.

 $\cos^2\theta$ = probability that v₁ contains the state v_e = $\cos^2(33^\circ)$ = 0.70. $\sin^2\theta$ = probability that v₂ contains the state v_e = $\sin^2(33^\circ)$ = 0.30.

$$\langle P(v_e \rightarrow v_e) \rangle_{\text{incoherent}} = 0.09 \cdot 0.70 + 0.91 \cdot 0.30 = 0.336$$

This is exactly what we observe in SNO and Super-K for ⁸B neutrinos!

"Survival probability" of a solar neutrino



The new generation of solar neutrino experiments



⁷Be: $E_v = 860 \text{ keV}$, monoenergetic line

Detection of solar neutrinos in BOREXINO: Elastic neutrino – electron scattering

$$v_x + e^- \rightarrow v_x + e^-$$
 (dominated by v_e)



via NC!BND School Rathen, 22+23.9.2009 26



BOREXINO

Expected (electron) energy distribution:





BOREXINO Detector



Mounting of Borexino photomultipliers with light concentrators







Installation of the 2 inner nylon spheres



Borexino during filling phase (scintillator in upper part, ultra pure water in lower part)



Scintillator filling completed May 15, 2007



Borexino: raw data, without cuts





Borexino: Spectrum after cuts





Borexino: result





49 ± 3stat ± 4sys cpd/100 t

Prediction of standard solar model:		
No oscillation	75 ± 5	cpd/100 t
Oscillation (LMA):		
"high metalicity"	48 ± 4	cpd/100 t
"low metalicity"	44 ± 4	cpd/100 t

"No-Oscillation" is excluded from BOREXINO alone with 4 sigma.

$$\phi(^{7}\text{Be}) = (5.18 \pm 0.51) \times 10^{9} \text{ cm}^{-2} \text{s}^{-1}$$

$$f(^{7}\text{Be}) = \phi(^{7}\text{Be}) / \phi_{\text{SSM}} = 1.02 \pm 0.10$$

$$f(\text{pp}) = 1.005^{+0.008}_{-0.020}$$

$$f_{\text{CNO}} < 3.8\%$$



Testing the MSW hypothesis:



Global analysis: Hints of $\theta_{13} > 0$

"Hints of θ13 > 0 from global neutrino data analysis", Fogli, Lisi, Marrone, Palazzo, Rotunno, arxiv:0806.2649v2

