Flavor Pendula

Part 0: General Introduction

Part I: Neutral Meson Mixing

Part II: 3-flavor Neutrino Mixing

Michael Kobel (TU Dresden)

22.9.09

BND school Rathen

Coupled Pendula

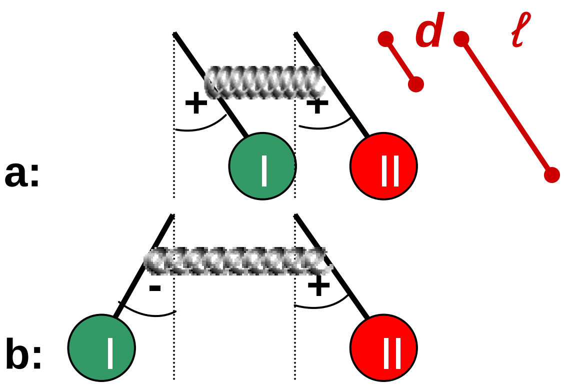
- Free Oscillation of one pendulum: $\omega_0^2 = \frac{g}{\ell}$
- 2 pendula with same length ℓ, mass *m* coupled by spring with strength *k*
- 2 Eigenmodes
  - Different eigenfrequencies = energies

    Mode a (II + I) with $\omega_a^2 = \omega_0^2$

    Mode b (II - I) with $\omega_b^2 = \omega_0^2 + \Delta\omega^2$
  - **Frequency (=energy) difference increases with stronger coupling**

$$\Delta\omega^2 = \frac{\boldsymbol{k}\boldsymbol{d}^2}{\boldsymbol{m}\ell^2}$$

  - Coupling can be steered by varying ***k*** or ***d*** (*we'll vary d in the following*)



Two bases in Hilbert-space

flavor-basis

- eigenstates of flavor
- eigenstates of weak charge
- *particles take part in weak interactions as flavor-eigenstates*

- Examples:
  - $\overline{K}^0(s\,\overline{u})$ or $K^0(\overline{s}\,u)$
  - ν_e, ν_μ, ν_τ

mass-basis

- eigenstates of mass
- well-defined lifetime
- *Particles propagate through space-time as mass-eigenstates*
  $$|\upsilon(t)\rangle = |\upsilon\rangle e^{i(\vec{p}\vec{x}-Et)} e^{-\Gamma t}$$

- Examples:
  - K^0_L, K^0_S
  - ν_1, ν_2, ν_3

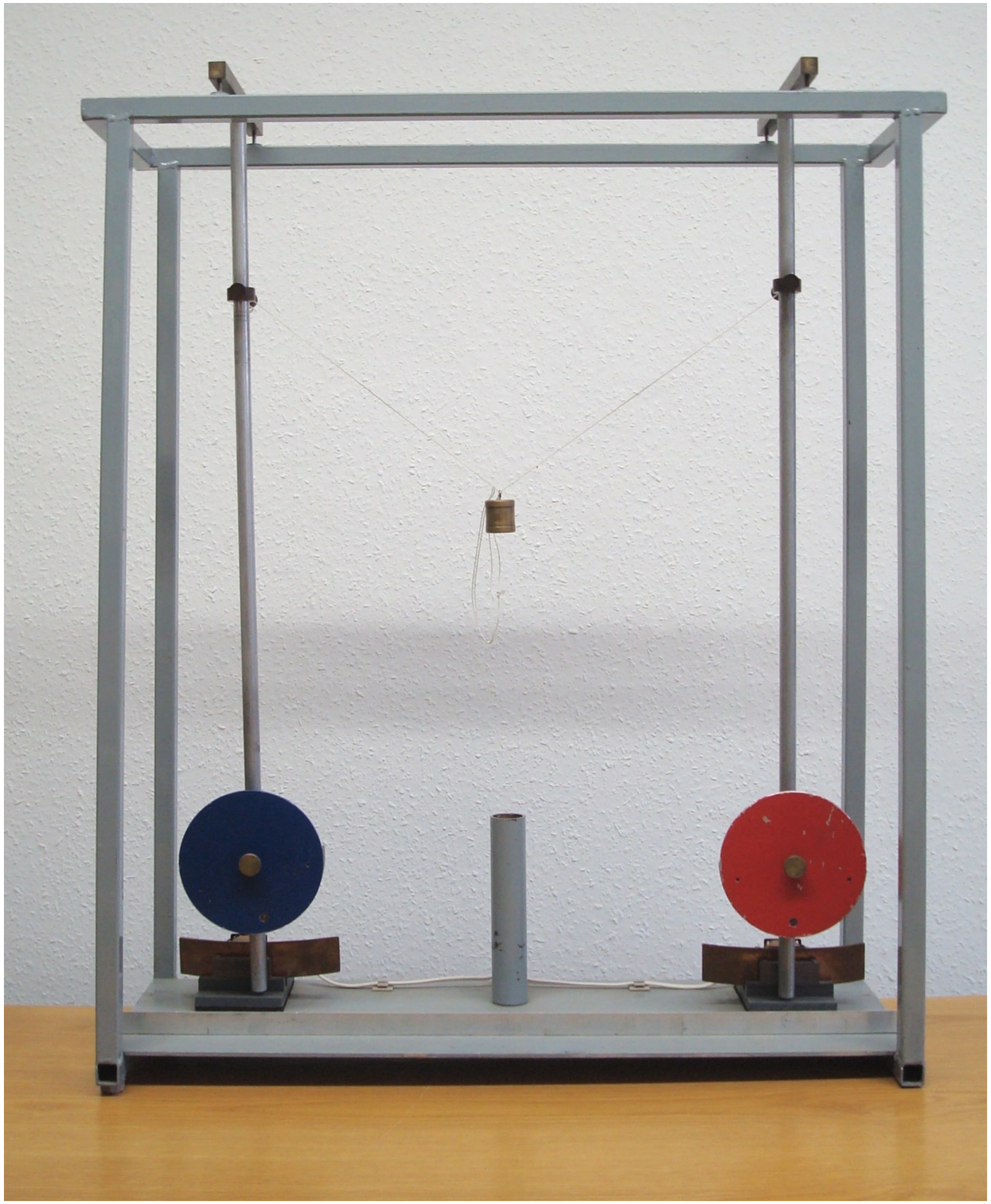
- Like coupled pendula, the coupling of particles leads to eigenstates with different masses and lifetimes, e.g. for linear combination of 2 states:

$$\nu_a = (\nu_\tau + \nu_\mu)/\sqrt{2} \quad \text{with} \quad m_a^2 = m_0^2$$
$$\nu_b = (\nu_\tau - \nu_\mu)/\sqrt{2} \quad \text{with} \quad m_b^2 = m_0^2 + \Delta m^2$$

Correspondences

pendulum	**particles**
Linear oscillation	complex phase rotation
Eigenmodes → fixed eigenfrequencies	**Mass eigenstates** → fixed phase frequencies
Frequency differences $\Delta\omega$ → different energies	Frequency differences $e^{i\Delta Et} \sim e^{i\Delta m^2 t}$ → different masses
One pendulum = lin. combination of eigenmodes	**Flavor eigenstate =** lin. combination of mass eigenstates
$\lvert amplitude^2\rvert \sim$ total energy in oscillation	$\lvert amplitude^2\rvert \sim$ detection probability
Beat-Frequency $\sim\Delta\omega$ of eigenmodes	**Flavor-Oscillation** $\sim \Delta m^{(2)}$ of mass eigenstates

Part I: Pendula for Neutral Meson Mixing



coupled pendula for demonstrating KK-, DD- and BB-mixing

Idea: Klaus Schubert
built 1987 in
Institut für Hochenergiephysik
Universität Heidelberg
at the occasion of the discovery
of BB-mixing by the
ARGUS-Collaboration at DESY

Part I: Neutral Meson Mixing

	u	c	t
$\overline{u}$	$\times$	D^0	$\diamond$
$\overline{c}$	$\overline{D^0}$	$\times$	$\diamond$
$\overline{t}$	$\diamond$	$\diamond$	$\times$

	d	s	b
$\overline{d}$	$\times$	K^0	B^0
$\overline{s}$	$\overline{K^0}$	$\times$	B_s
$\overline{b}$	$\overline{B^0}$	$\overline{B_s}$	$\times$

- Need to be neutral and have distinct anti-particle (x)
- Needs to have a non-zero lifetime
  - top is so heavy, it decays long before it can even form a meson ($\diamondsuit$)
- That leaves four distinct cases...

Some sheets stolen from Gerhard Raven's

Note: for (much!) more detail, see eg. arXiv:hep-ex/0103016v1

Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t)=\begin{pmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*} & M-\frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Solution (in terms of eigenvectors):

$$\psi(t) = a\,|B_H(t)\rangle + b\,|B_L(t)\rangle$$

(a and b determined by initial conditions)

Eigenvectors:

$$|B_H\rangle = p|B\rangle + q|\overline{B}\rangle$$

$$|B_L\rangle = p|B\rangle - q|\overline{B}\rangle$$

Evolution of eigenvectors:

$$|B_H(t)\rangle \;=\; |B_H\rangle\, e^{-i\left(M+\frac{1}{2}\Delta m-\frac{i}{2}(\Gamma-\Delta\Gamma)\right)t}$$

$$|B_L(t)\rangle \;=\; |B_L\rangle\, e^{-i\left(M-\frac{1}{2}\Delta m+\frac{i}{2}(\Gamma+\Delta\Gamma)\right)t}$$

From the eigenvector calculation:

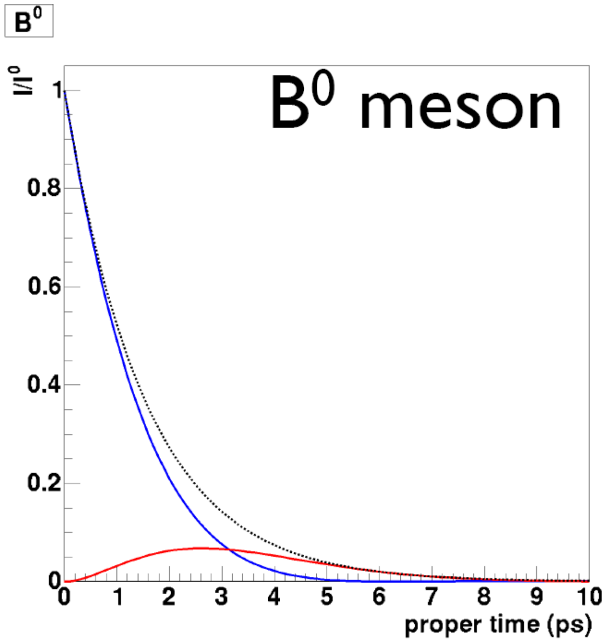
$$\frac{q}{p}=\sqrt{\frac{M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*}}{M_{12}-\frac{i}{2}\Gamma_{12}}}$$

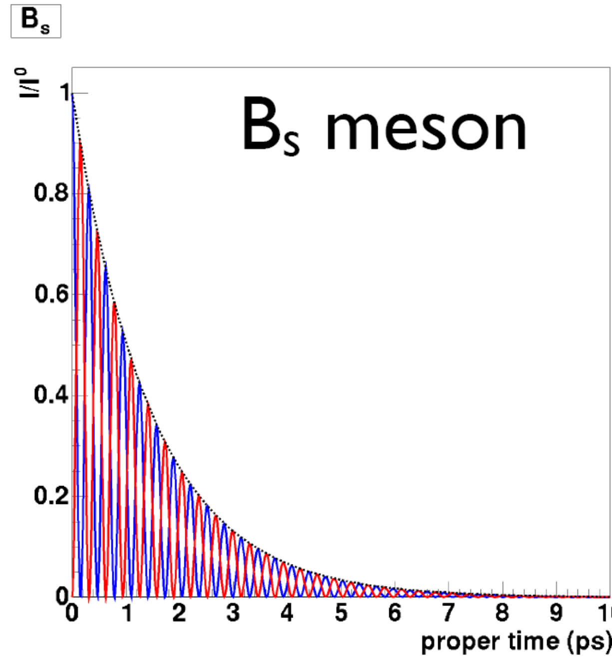
Δm and $\Delta\Gamma$ follow from the eigenvalues:

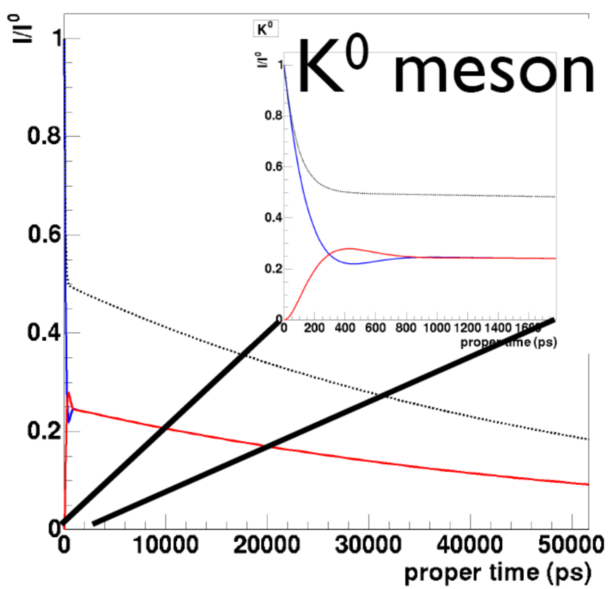
$$\Delta m+\frac{i}{2}\Delta\Gamma = 2\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*}\right)}$$

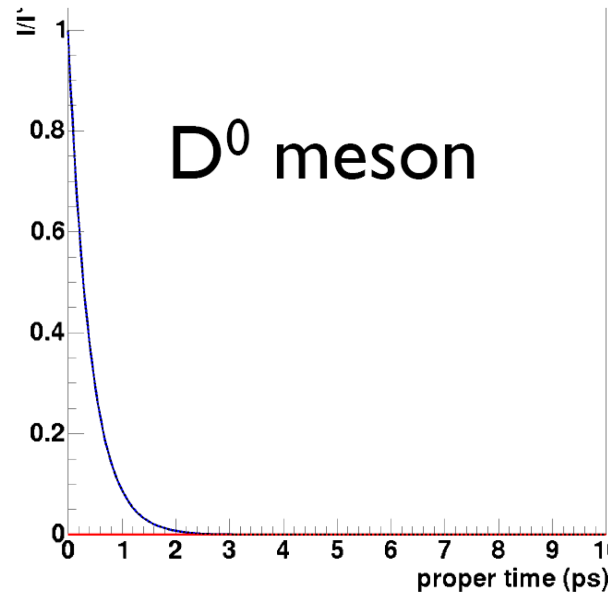
if: $\Gamma_{12}=0 \Rightarrow \Delta\Gamma=0, \left|\frac{q}{p}\right|=1$

Summary of Neutral Meson Mixing









	$x = \frac{m_1 - m_2}{(\Gamma_1 + \Gamma_2)/2}$	$y = \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$
B^0	**0.75**	**~0.01**
B_s	**26**	~0.10
K^0	0.95	**0.996**
D^0	**0.009**	0.007

Blue:
given a P^0, at t=0,
the probability of
finding a P^0 at t.

Red:
given a P^0, at t=0,
the probability of
finding a P^0bar at t.

How can pendula model all this?

→*Just start with **one** pendulum (e.g. $\bar{B}^0$ or B^0)!*

- $\bar{B}^0(b\,\bar{d}) - B^0(\bar{b}\,d)$ oscillations:
  - very few decay channels common to both
    → *Each flavor is damped separately*
  - **Just few flavor-oscillation periods observable**

- $\bar{B}_s^0(b\,\bar{s}) - B_s^0(\bar{b}\,s)$ oscillations:
  - mixing via $V_{ts}^2 = 0.04^2 > V_{td}^2 = 0.009^2$ much faster compared to B^0
    → *well, model it by slower decay relative to mixing…*
  - **many flavor-oscillation (beat) periods observable**

- $\bar{D}^0(u\,\bar{c}) - D^0(\bar{u}\,c)$ oscillations:
  - mixing via $(m_b, m_s) << m_t$ much slower *compared to B^0*
    → *again, model it by faster decay relative to mixing…*
  - **no flavor-oscillation period observable**

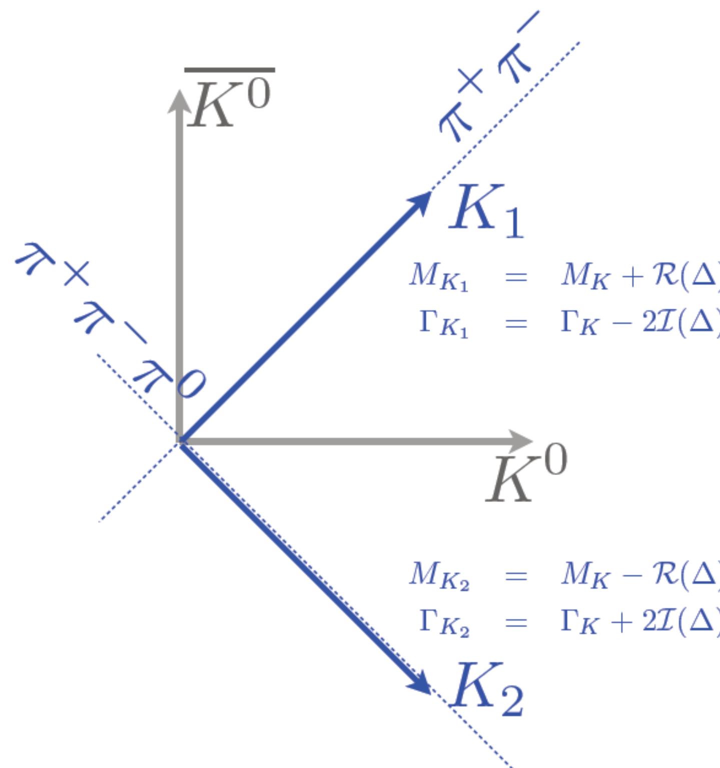
- $\bar{K}^0(s\,\bar{d}) - K^0(\bar{s}\,d)$ oscillations:
  - most decay channels in common ($\pi\pi$, $\pi\pi\pi$)
    → *The damping is in the coupling!*
  - Large Phase space difference for decays of mass eigenstates (CP-cons.!)
    → *The damping has to be very different for the eigenmodes*
  - **After a while only the K_L^0 eigenmode remains**

Neutral Kaon Mixing

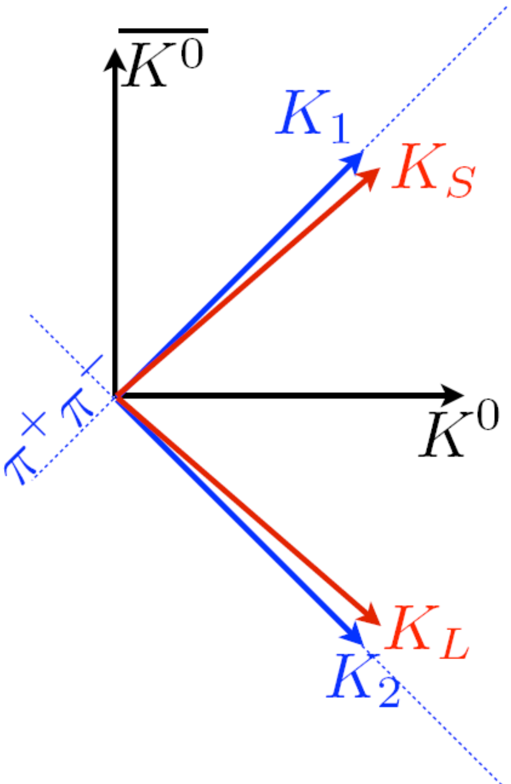
$$|K_1\rangle = \frac{|K^0\rangle + |\overline{K^0}\rangle}{\sqrt{2}}$$

$$|K_2\rangle = \frac{|K^0\rangle - |\overline{K^0}\rangle}{\sqrt{2}}$$

- K_1 and K_2 are their *own* antiparticle, but one is CP even, the other CP odd
- *Only* the CP even state can decay into 2 pions
  - $|K_1\rangle$ (CP=+1) → ππ (CP=-1 * -1 =+1)
- The CP odd state will decay into 3 pions instead
  - $|K_2\rangle$ (CP=-1) → ππ π (CP = -1*-1*-1 = -1)
- <u>There is a huge difference in available phasespace between the two (~600x!) → the CP *even* state will decay much faster</u>
  - Difference due to M(K^0) ≅ 3M(π)
  - Δ has a large imaginary component!



What about CP-violation?



EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey

(Received 10 July 1964)

three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as $K_2^0 = 2^{-1/2}[(K_0-\bar{K}_0)+\epsilon(K_0+\bar{K}_0)]$ then $|\epsilon|^2 \cong R_T\tau_1\tau_2$ where τ_1 and τ_2 are the K_1^0 and K_2^0 mean lives and R_T is the branching ratio including decay to two π^0. Using $R_T = \frac{3}{2}R$ and the branching ratio quoted above, $|\epsilon| \cong 2.3\times 10^{-3}$.

$$|K_L\rangle = p\left|K^0\right\rangle - q\left|\overline{K^0}\right\rangle$$

$$|K_S\rangle = p\left|K^0\right\rangle + q\left|\overline{K^0}\right\rangle$$

Have a *choice* when 'parameterizing' K_S and K_L:

1. in terms of K^0 and $\overline{K^0}$
2. in terms of K_1 and K_2

Historically, 'kaon physics' has chosen 2, but in in 'B physics' (next lectures!), the equivalent of 1 is very much dominant (as ε_B=0, but still $q \neq 1$ & $p \neq 1$...)

$$\langle K_L | K_L\rangle \equiv 1 \Rightarrow |q|^2 + |p|^2 = 1$$

eg. $$\begin{aligned} p &= 1+\epsilon \\ q &= 1-\epsilon \end{aligned} \quad \text{with } |\epsilon| << 1$$

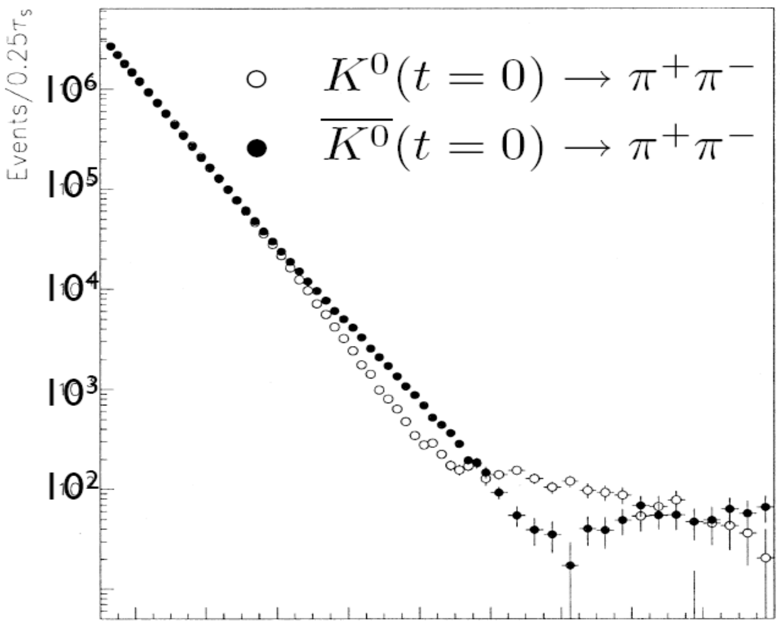
This tends to be *very* confusing...

CP-violation with pendula

- Realisation: differences between the $\overline{K}^0$ and K^0 pendula
  - Different moments of inertia $m\ell^2$ (m and/or ℓ different)
  - ***Different coupling strengths (lever arm d different)***
    → *corresponds to differences of V_{td} and V_{td}* in mixing diagram*

- K_L has unequal amounts of $\overline{K}^0$ and K^0
  → *wait for K_L and have a look!*

- Equal amounts of $\overline{K}^0$ and K^0 evolve differently
  → *former eigenmodes now develop!*
  (cf. CPLear Experiment)



Is **CP** violation with pendula a perfect model?

- No!
  - all mechanical devices are **T** invariant!
  - if **CP** is violated and **T** is conserved:
    → **CPT** is violated!
- In fact, we have modelled CPT violation!

Part II: Neutrino flavor pendula

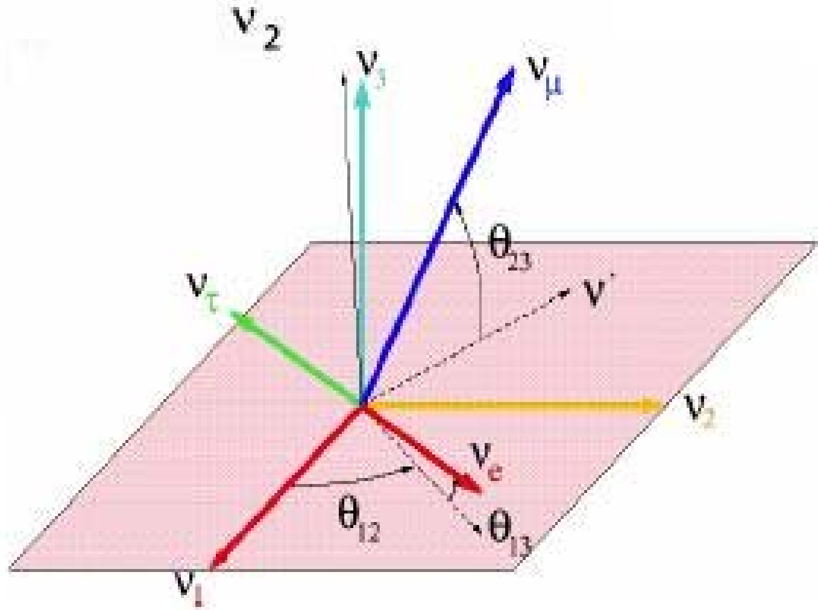
coupled pendula for demonstrating 3-flavor neutrino mixing as realized in nature

Idea: Michael Kobel
built 2004 at Uni Bonn,
extended 2006 at TU Dresden
with variable mixing angles
and digital readout
http://neutrinopendel.tu-dresden.de

3-flavor neutrino mixing

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$\Theta_{\text{atmos, beam}}$ θ_{13}, δ $\Theta_{\text{solar, reactor}}$

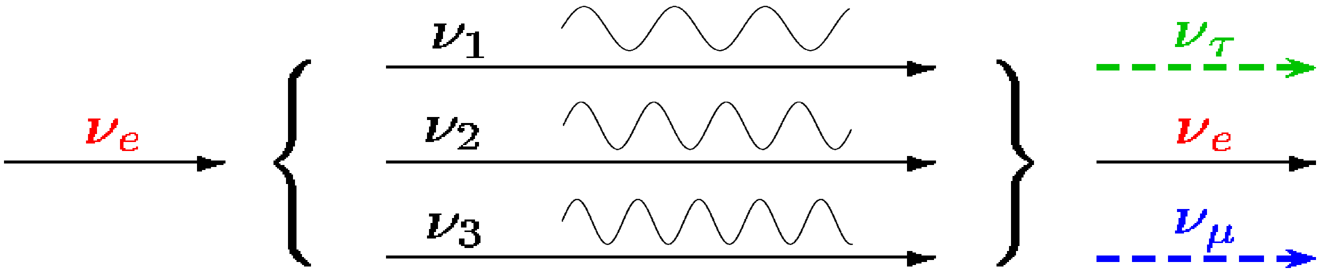


PMNS mixing matrix
(w/o Majorana Phases)

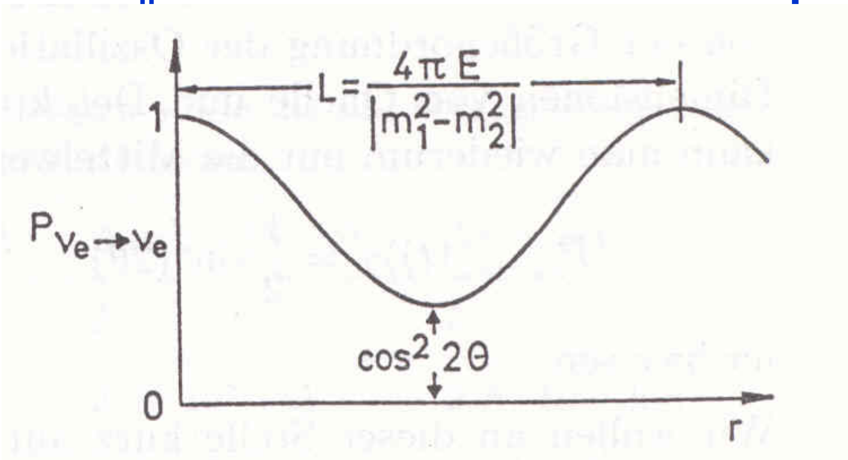
- 3 Mixing angles: θ_{12}, θ_{23}, θ_{13}
- 1 CP-violating Dirac-Phase: δ

ν flavor-oscillations

- Each flavor (e.g. ν_e) is sum of mass eigenstates (ν_1, ν_2, ν_3)
- Each mass eigenstate with fixed p has a different phase frequency ω_i
  - $exp(i\omega_i t) = exp(iE_i t) = \exp(i(\sqrt{(p^2+m_i^2)}t) \sim exp(ipt+\mathbf{im_i^2t}/2p+\ldots)$



- **The differences $\Delta\omega_{ij} \sim |m_i^2 - m_j^2| =: \Delta m_{ij}^2$ lead to flavor oscillations**
  - Δm_{ij}^2 determines the oscillation **period**
  - θ_{ij} determines the oscillation **amplitude**



$$L_{ij} = 2.5m\frac{E(MeV)}{\Delta m_{ij}^2(eV^2)}$$

Current values

cf. global fit Th.Schwetz et al., NJP 10 (2008)

Δm^2_{23} = 2,4 x 10^{-3} eV^2	**Δm^2_{13} = 2,5 x 10^{-3} eV^2**	**Δm^2_{12} = 0,08 x 10^{-3} eV^2**
„fast“ oscillation		„slow“ oscillation
$L_{23} = 1\,km \times E(MeV)$		$L_{12} = 30\,km \times E(MeV)$
$\theta_{23} = 45° \pm 3°$	$\theta_{13} < 11°$ (90% CL)	$\theta_{12} = 33.5° \pm 1.5°$

$\Theta_{\text{atmos, beam}}$ $\quad$ θ_{13}, δ $\quad$ $\Theta_{\text{solar, reactor}}$

- consistent with so-called tri/bi-maximal mixing

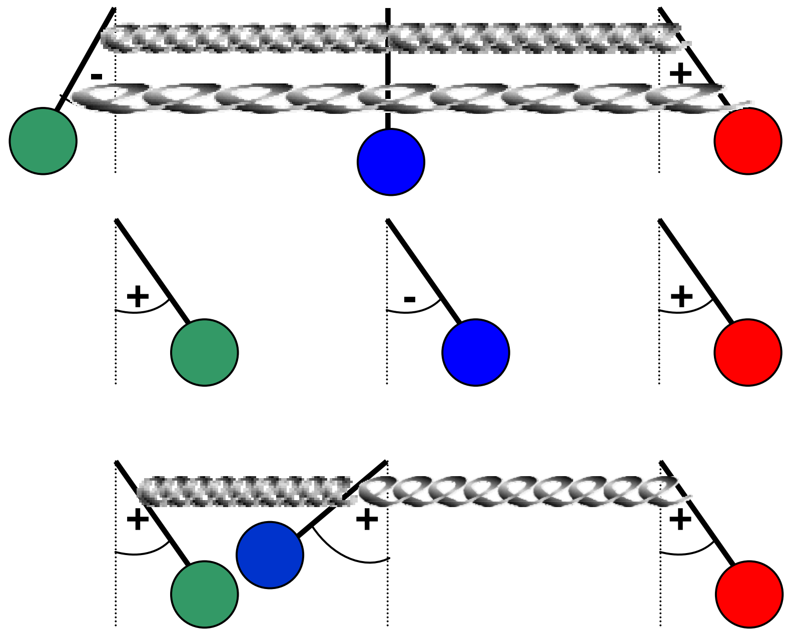
$\theta_{23} = 45°$ $\quad$ $\theta_{13} = 0°$ $\quad$ $\theta_{12} = 35.3°$

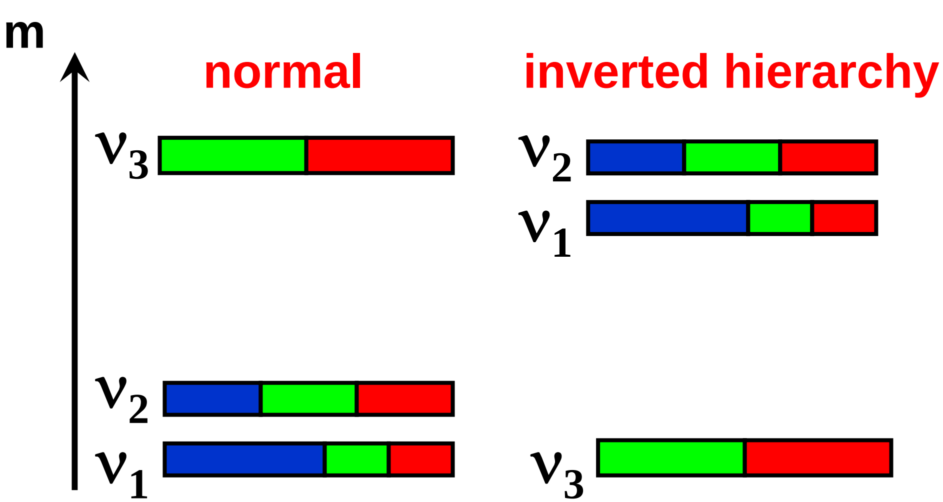
$$U_{PMNS} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

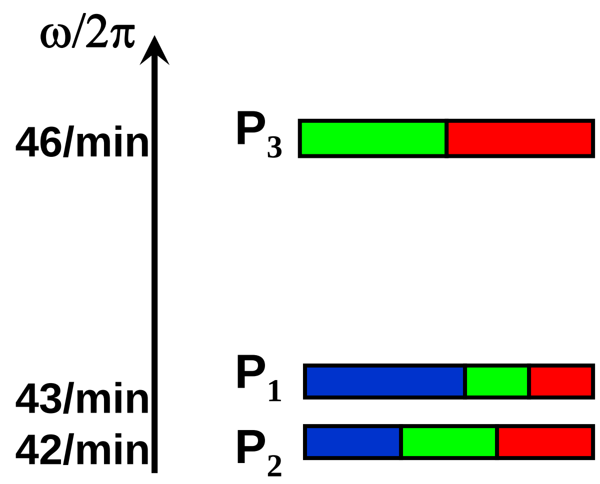
Harrison, Perkins, Scott ’99,’02
Z.Xing,’02, He, Zee, ’03, Koide ’03
Chang, Kang, Kim ’04, Kang ’04

Realisation as coupled pendula

- $\nu_3 = (\qquad - \nu_\mu + \nu_\tau)/\sqrt{2}$
- $\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$
- $\nu_1 = (2\nu_e + \nu_\mu + \nu_\tau)/\sqrt{6}$







The solar neutrino „deficit“

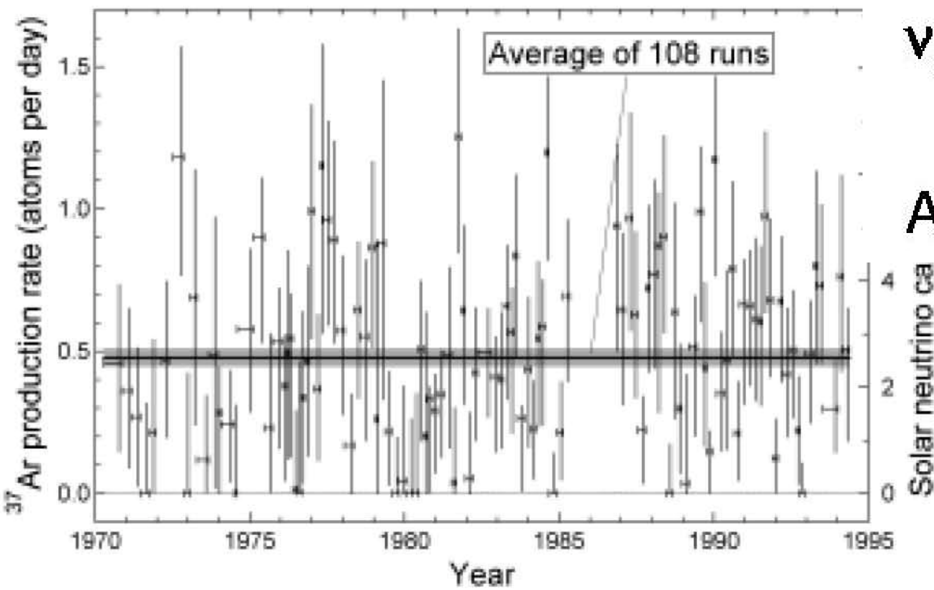
Ray Davis

Nobelpreis 2002

380000 l
Perchlorethylen
in der Homestake- Mine

$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$

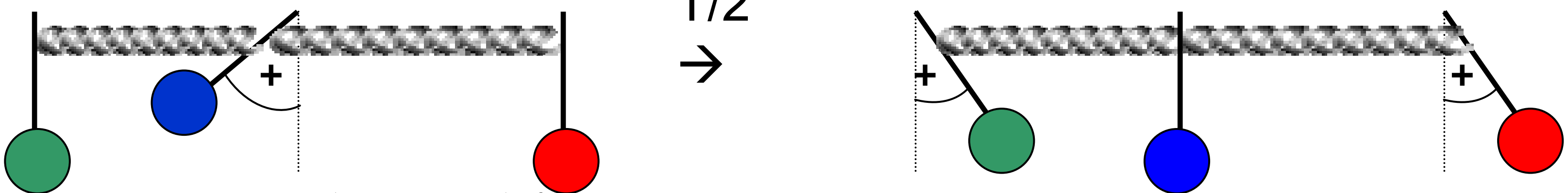
Ausspülen des ^{37}Ar (0.5 Atome/Tag)



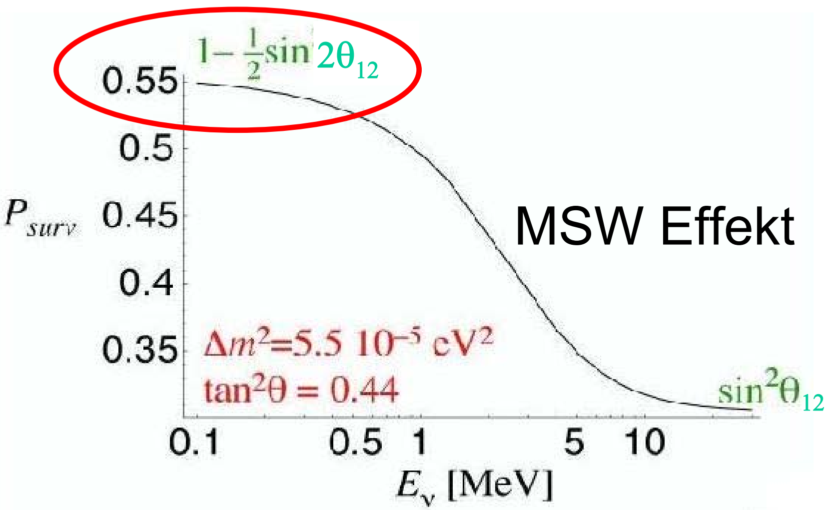
- Davis: only sensitiv to ν_e
  Result: Only 30% of expected ν_e detected

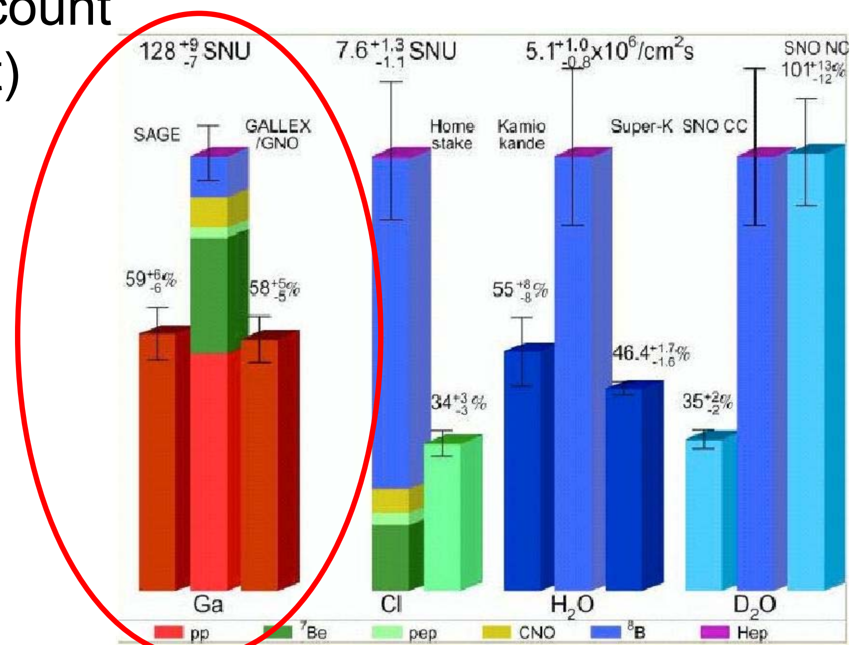
need for enhancement (MSW effect)

- nuclear fusion: 100% ν_e leave the sun (w/o MSW effect)
  $4p \rightarrow {}^4He + 2e^+ + 2\nu_e + 27$ MeV
- “slow“ oscillation via Δm^2_{12} and θ_{12}, pendula: weak coupling

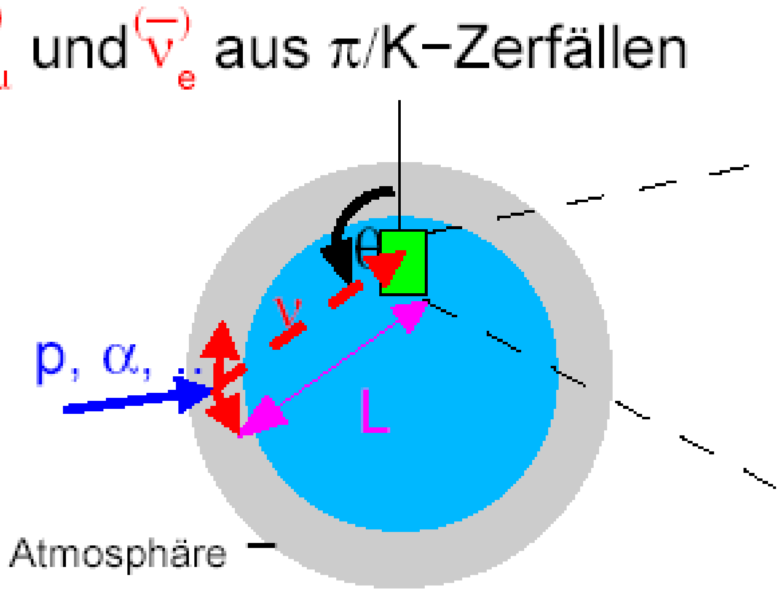


- transition to $(\nu_\tau - \nu_\mu)/\sqrt{2}$ not possible, since ν_e not in ν_3
- oscillation only to $(\nu_\tau + \nu_\mu)/\sqrt{2}$
- $P(\nu_e \rightarrow \nu_e) > 50\%$ since just ν_1 and ν_2 count
  → need for enhancement (MSW effect)

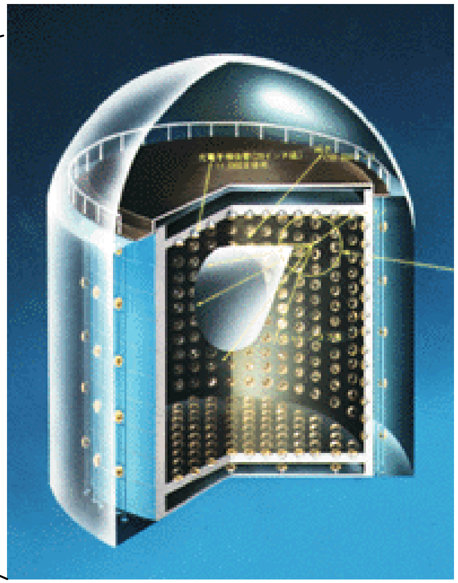


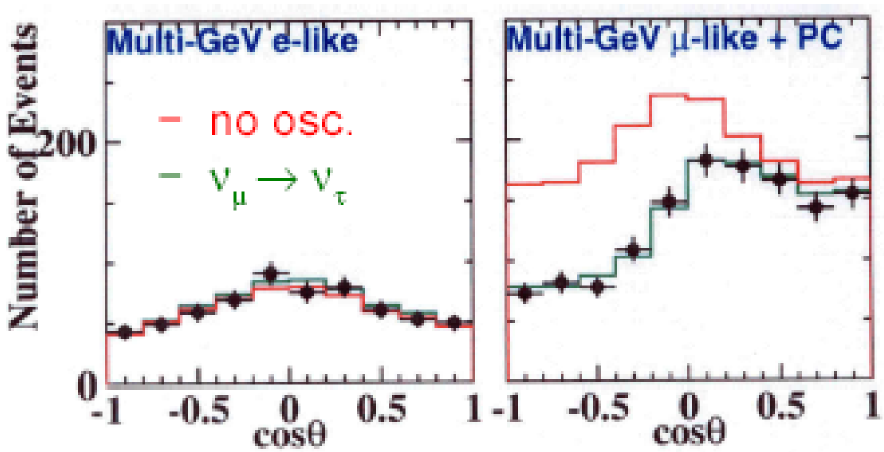


atmospheric neutrinos



SuperKamiokande 2000:





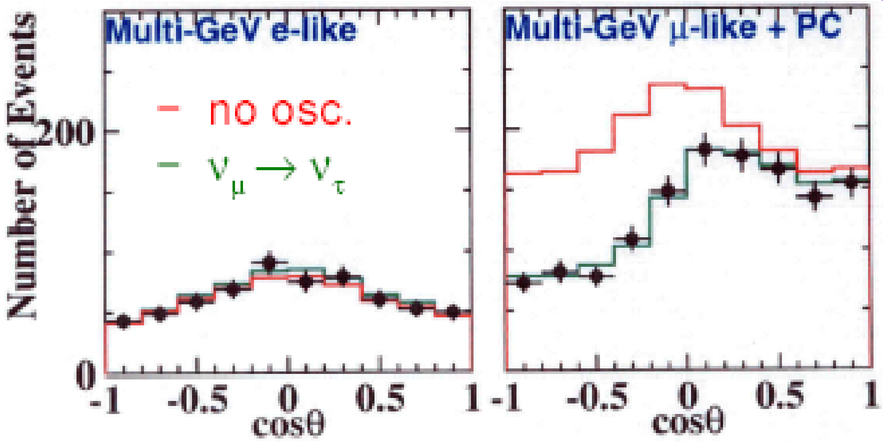
(C. Mc Grew, NOON 2000, Dez.2000)

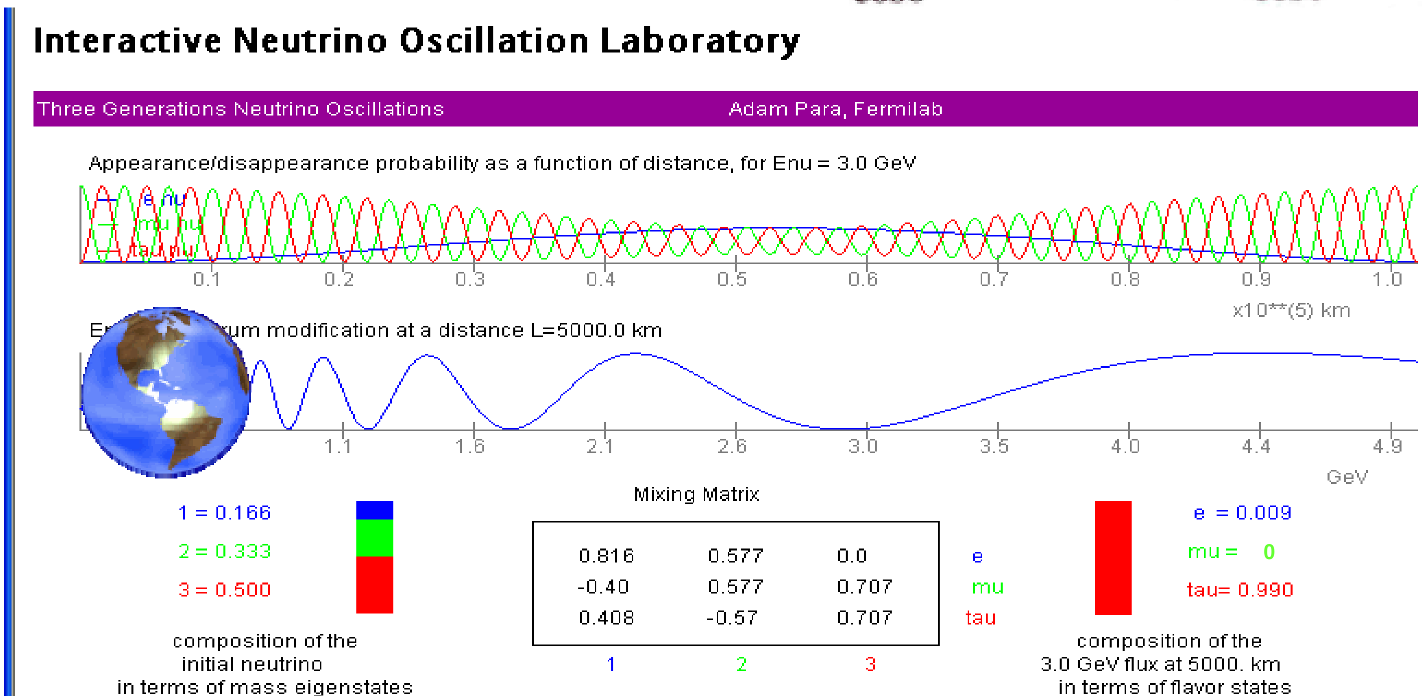
look at ν_e and ν_μ from air showers:

- no deficit for ν_e
- clear deficit for ν_μ
- fully compatible with $\nu_\mu \rightarrow \nu_\tau$

atmospheric neutrinos

- SuperKamiokande 2000: described als $\nu_\mu \rightarrow \nu_\tau$
- pendula:
  ν_e : weak coupling to ν_μ, ν_τ
  ν_μ: weak coupling to ν_e
  strong coupling to ν_τ

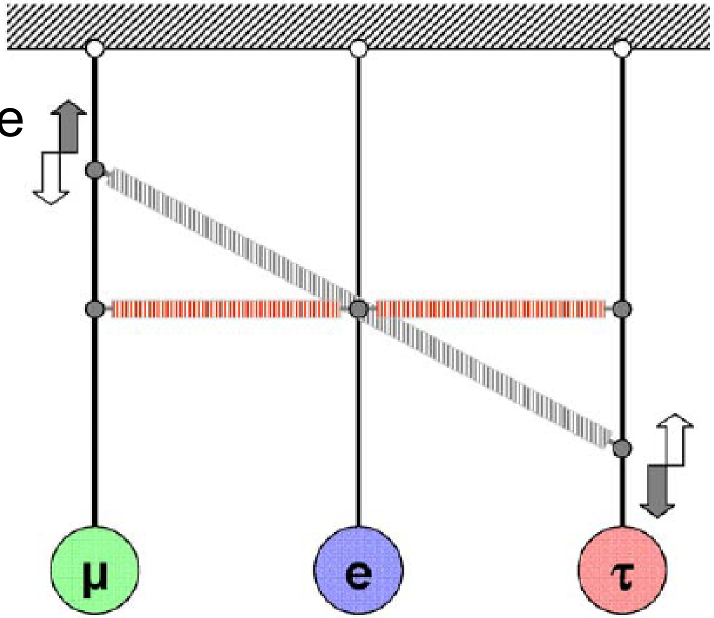




http://minos.phy.bnl.gov/nu-osc-lab/Superposition1.html

Modify θ_{23}

- Non-maximal mixing of ν_μ and ν_τ
- $\nu_3 = (\qquad - \nu_\mu + \nu_\tau)/\sqrt{2}$ no longer eigenmode



Possible range:
$30^o < \theta_{23} < 60^o$

θ_{23} smaller
θ_{23} larger

- http://neutrinopendel.tu-dresden.de
(special high school thesis J. Pausch 2008)

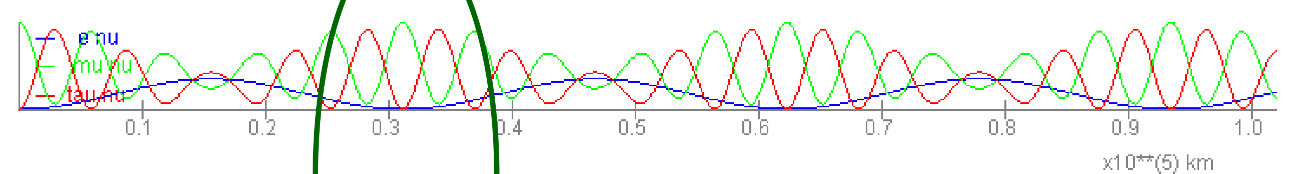


Abbildung 52: Atmosphärische Neutrino-Oszillation mit θ_{23} =0,9rad (51,6°), Java-Applet.

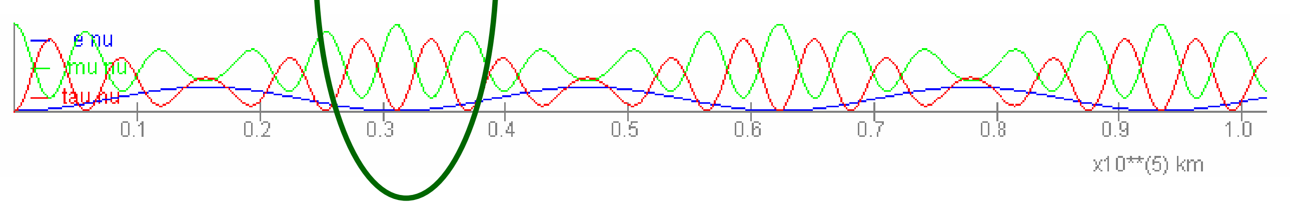


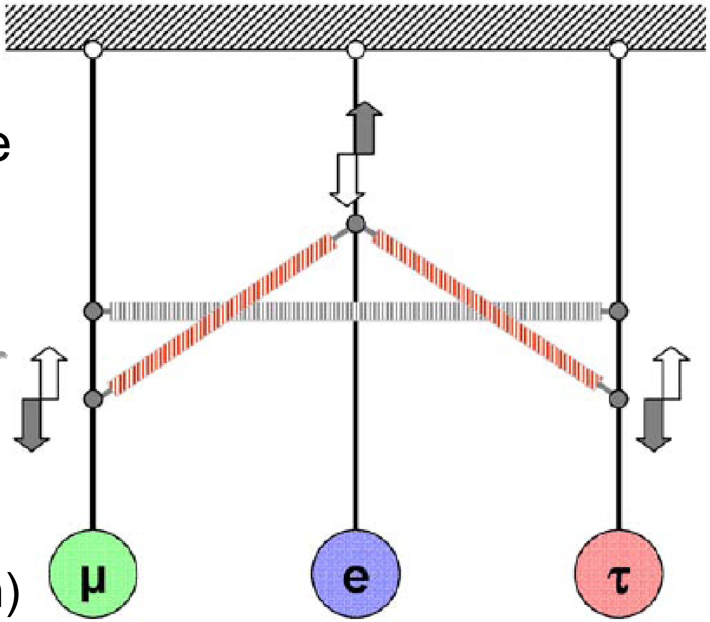
Abbildung 53: Atmosphärische Neutrino-Oszillation mit =0,98rad (56,2°), Java-Applet.

Modify θ_{12}

- Modify fraction of ν_e in ν_1 and ν_2
- $\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$ no longer eigenmode

Possible range:
$20^o < \theta_{12} < 55^o$

θ_{12} smaller
θ_{12} larger



- http://neutrinopendel.tu-dresden.de (J. Pausch)

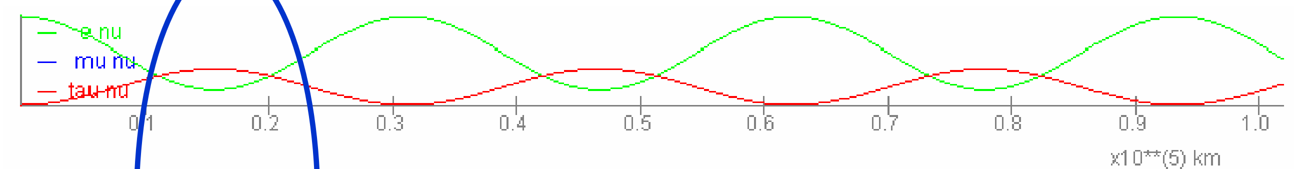


Abbildung 27: Sonnenneutrino-Oszillation mit θ_{12}=0,57rad (33°), Java-Applet.

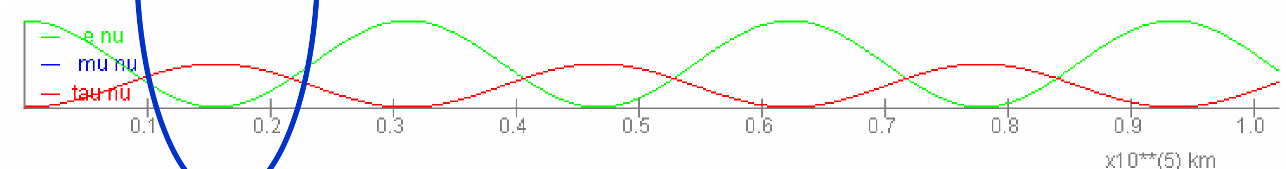


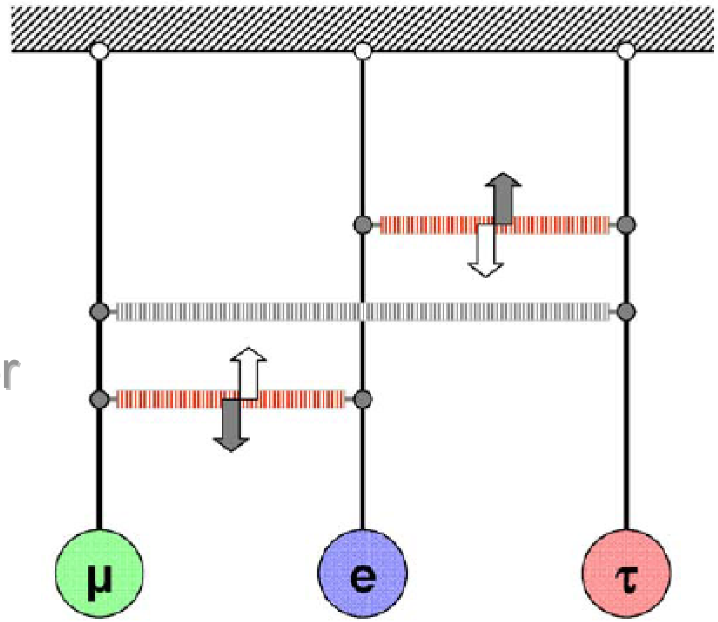
Abbildung 28: Sonnenneutrino-Oszillation mit θ_{12}=0,7854rad (45°), Java-Applet.

Modify θ_{13}

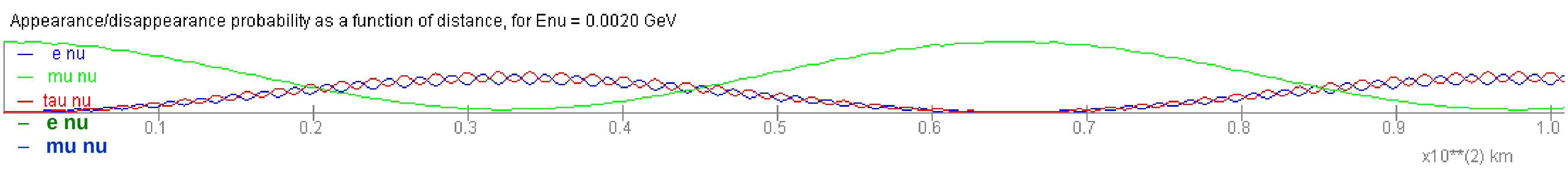
- ν_e present in $\nu_3 \sim (\sin\theta_{13}\nu_e - \nu_\mu + \nu_\tau)$
- ν_e can now excite $(\nu_\tau - \nu_\mu)$ mode, inducing fast $\nu_\tau - \nu_\mu$ modulation

Possible range:
$-6^o < \theta_{13} < 6^o$

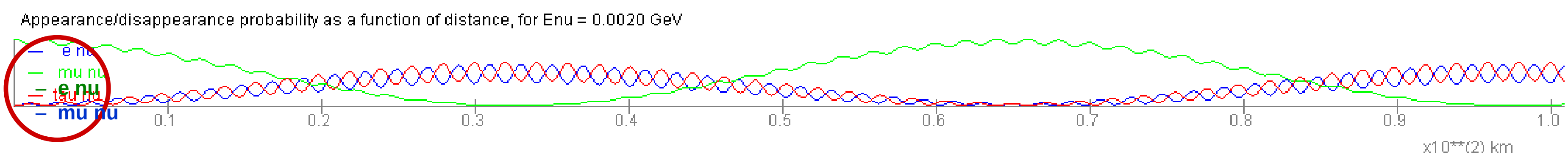
θ_{13} smaller
θ_{13} larger



- Reactor neutrinos (2 MeV)
  - $\sin\theta_{13} = 0.10 \quad (\theta_{13} = 6^o)$

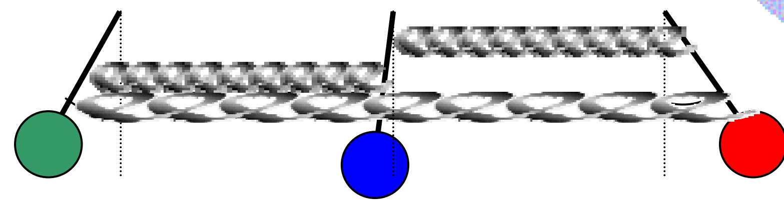


  - $\sin\theta_{13} = 0.20 \quad (\theta_{13} = 12^o)$



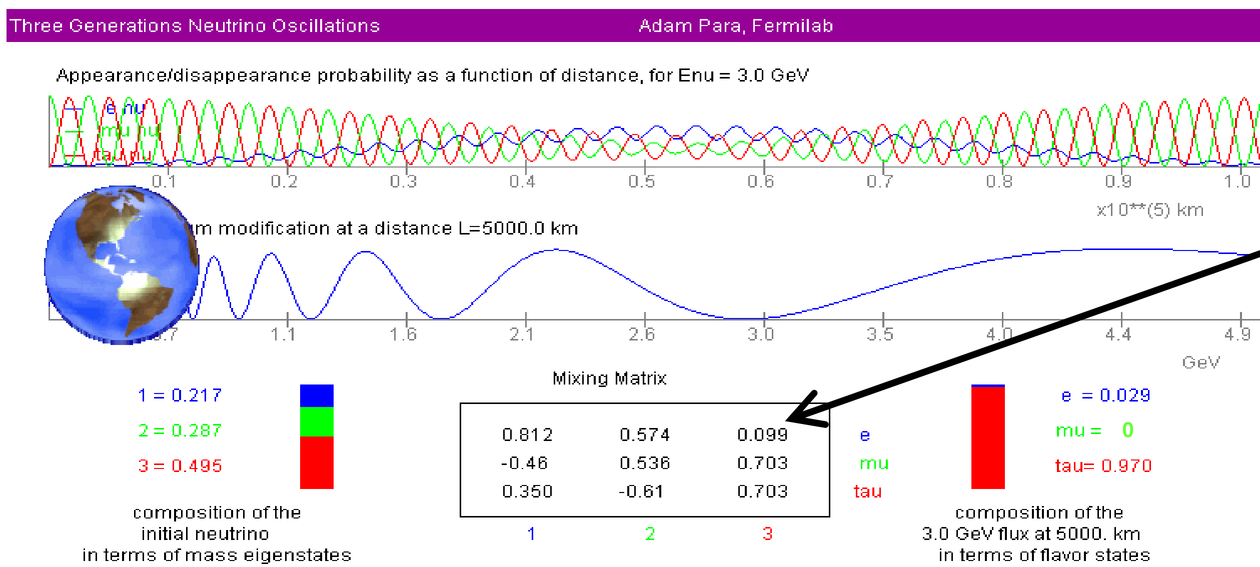
Impact of θ_{13} on atmospheric ν

- $\nu_3 = (\sin\theta_{13}\nu_e - \nu_\mu + \nu_\tau)/\sqrt{2.01}$



- reactor $\bar{\nu}_e \rightarrow \bar{\nu}_\tau + \bar{\nu}_\mu$ disappearance and atmospheric $\nu_\mu \rightarrow \nu_e$ appearance
  - „slow“ *directly* via Δm_{12} (weak coupling)
  - „fast“ modulation via $\nu_\tau - \nu_\mu$ with Δm_{23} (strong coupling)

Interactive Neutrino Oscillation Laboratory



$\theta_{13} = 6^o$

$\sin\theta_{13} = 0.1$

$\sin^2 2\theta_{13} = 0.04$

Are neutrino pendula a perfect model?

- Few “features”
  - Need “creative” sign convention, leading to
  - imperfection for understanding sequence of masses
  - imperfection for $\theta_{23} \neq 45^o$
    - some $(\nu_\tau - \nu_\mu)$ present in ν_1 and ν_2
    - but $\nu_e \rightarrow (\nu_\tau - \nu_\mu)$ still not possible!
- Else perfect!

The END !