

Dark Matter Lecture 2: Principles of direct WIMP detection

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Direct Detection of WIMPs: principle



- Elastic collision between WIMPs and target nuclei
- The recoil energy of the nucleus is:

$$E_{R} = \frac{|\vec{q}|^{2}}{2m_{N}} = \frac{\mu^{2}v^{2}}{m_{N}}(1 - \cos\theta)$$

- q = momentum transfer $|\vec{q}|^2 = 2\mu^2 v^2 (1 \cos\theta)$
- μ = reduced mass (m_N = nucleus mass; m_X = WIMP mass)

$$\mu = \frac{m_{\chi}m_{N}}{m_{\chi} + m_{N}}$$

• v = mean WIMP-velocity relative to the target

• θ = scattering angle in the center of mass system

Expected Rates in a Detector

- For now strongly simplified: Astrophysics $R \propto N \frac{\rho_{\chi}}{m_{\chi}} \sigma_{\chi N} \cdot \langle \mathbf{v} \rangle$ Particle physics
- N = number of target nuclei in a detector
- ρ_{χ} = local density of the dark matter in the Milky Way
- <v> = mean WIMP velocity relative to the target
- m_{χ} = WIMP-mass
- σ_{xN} =cross section for WIMP-nucleus elastic scattering

Local Density of WIMPs in the Milky Way



Expected Rates in a Detector

• The differential rate (still strongly simplified) is:

$$\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-\frac{E_R}{E_0 r}}$$

- **R** = event rate per unit mass
- E_R = nuclear recoil energy
- **R**₀ = total event rate
- **E**₀ = most probable energy of WIMPs



• **r** = kinematic factor

$$=\frac{4m_{\chi}m_{N}}{\left(m_{\chi}+m_{N}\right)^{2}}$$

r



$$\int_0^\infty \frac{dR}{dE_R} dE_R = R_0$$

$$\langle E_R \rangle = \int_0^\infty E_R \frac{dR}{dE_R} dE_R = E_0 r$$

Some Typical Numbers

• We assume that the WIMP mass and the nucleus mass are identical:

$$m_{\chi} = m_N = 100 \ GeV \cdot c^{-2}$$

$$\Rightarrow r = \frac{4m_{\chi}m_{N}}{\left(m_{\chi} + m_{N}\right)^{2}} = 1 \qquad \text{kinematic factor}$$

 $v \sim 220 \text{ km s}^{-1} = 0.75 \times 10^{-3} c$

mean WIMP velocity relative to target (halo is stationary, Sun moves through halo)

$$\langle E_R \rangle = E_0 = \frac{1}{2} m_{\chi} v^2$$
$$\langle E_R \rangle = \frac{1}{2} 100 \frac{GeV}{c^2} (0.75 \times 10^{-3} c)^2$$
$$\langle E_R \rangle \approx 30 \text{ keV} \qquad \text{mean recoind}$$

mean recoil energy deposited in a detector

Expected Rates in a Detector

- We have to take into account following facts:
 - The WIMPs will have a velocity distribution f(v)
 - the detector is on Earth, which moves around the Sun, which moves around the galactic center
 - the cross section depends on whether the interaction is spin-independent (SI), or spin-dependent (SD)
 - the WIMPs scatter on nuclei, which have a finite size; we have to consider form-factor corrections < 1 (different for SI and SD interactions)
 - the nuclear recoil energy is not necessarily the observed energy, since in general the detection efficiency is < 1</p>
 - detectors have a certain energy resolution and energy threshold



Kinematics

• WIMPs with velocity v and incident kinetic energy $E_i = \frac{1}{2}m_{\chi}v^2$ which are scattered under an angle θ in the center of mass system, will yield a recoil energy E_R in the laboratory system:



Kinematics

- Assumption: the scattering is isotropic => uniform in cos(θ)
- An incoming WIMP with energy E_i will deliver recoil energies uniformly in:

$$0 \le E_R \le E_i r$$

• We had looked at the case with r = 1 (equal masses), a stationary target and $\theta = 180^{\circ}$ (head-on collision)

$$E_R = E_i$$

- How does the overall spectrum look like? We will sample the incident spectrum.
 - In each interval $E_i \to E_i + dE_i$ we will have a contribution to the spectrum in $E_R \to E_R + dE_R$ at rate $dR(E_i)$ of $dR(E_i)$





Kinematics

• We have to integrate over all incoming WIMP energies:

$$\frac{dR}{dE_R}(E_R) = \int_{E_{\min}}^{E_{\max}} \frac{dR(E_i)}{E_i r}$$

- For E_{max} : we will use as maximum speed either ∞ or v_{esc} (we will discuss this later)
- For E_{min}: to deposit a certain recoil energy E_R, we need an incident WIMP energy:

$$E_i \ge \frac{E_R}{r} \equiv E_{\min}$$

• We will now determine the differential rate.



Coordinate System



Maxwell-Boltzmann velocity distribution

• The event rate per unit mass in a detector with nuclear mass number A is:

$$dR = \frac{N_A}{A}\sigma \, \mathrm{v} \, dn$$

 \Rightarrow N_A =6.022×10²⁶ kg⁻¹ Avogadro number

 $rightarrow \sigma$ = cross section for the scattering on the nucleus

volume σ·v swept per unit of time contains dn(v) particles with velocity v

• The differential particle density dn is taken as a function of the velocity v:

$$dn = \frac{n_0}{k} f(\vec{v}, \vec{v}_E) d^3 \vec{v}$$

- with the mean WIMP number density $n_0 = \frac{\rho_{\chi}}{m_{\chi}}$ \Rightarrow v = velocity relative to the target (which is on Earth)
 - \mathbf{v}_{E} = Earth velocity (and thus target velocity) relative to the dark matter distribution

• k is a normalization constant, so that:

$$\int_0^{v_{esc}} dn \equiv n_0 \qquad \begin{array}{c} \text{mean WIMP} \\ \text{number density} \end{array}$$

- where $v_{esc} = \text{local galactic escape velocity}$ ($\approx 544 \text{ km/s}$)
- this means:

$$k = \int f(\vec{\mathbf{v}}, \vec{\mathbf{v}}_{\rm E}) d^3 \vec{\mathbf{v}}$$



$$k = \int_0^{2\pi} d\phi \int_{-1}^{+1} d(\cos\theta) \int_0^{v_{esc}} f(\vec{v}, \vec{v}_E) v^2 dv$$

• We assume a Maxwell-Boltzmann WIMP velocity distribution with respect to the galactic frame:

$$f(\vec{v}, \vec{v}_E) = e^{-\frac{(\vec{v} + \vec{v}_E)^2}{v_0^2}} \qquad \vec{v} + \vec{v}_E \qquad \text{WIMP velocity in the galaxy frame}$$

$$v_0 \approx 220 \text{ km s}^{-1}$$

- We first look at the simplified case of a stationary Earth v_{E} = 0 and v_{esc} = ∞
- For this case, we have:

$$k = k_0 = \int_0^{2\pi} d\phi \int_{-1}^{+1} d(\cos\theta) \int_0^{\infty} e^{-\frac{(\vec{v}+0)^2}{v_0^2}} v^2 dv = 4\pi \int_0^{\infty} e^{-\frac{(\vec{v})^2}{v_0^2}} v^2 dv = (\pi v_0^2)^{3/2}$$

• and thus

$$dR = R_0 \frac{1}{2\pi v_0^4} vf(v,0)d^3v$$

• with the total rate R_0 per unit mass ($V_E = 0$ and $V_{esc} = \infty$) being defined as:

$$R_0 = \frac{2}{\sqrt{\pi}} \frac{N_A}{A} \frac{\rho_{\chi}}{m_{\chi}} \sigma_0 \, v_0$$

• For a Maxwell-Boltzmann distribution

$$f(\mathbf{v},0) = e^{-\frac{\mathbf{v}^2}{\mathbf{v}_0^2}}$$

- isotropic: $d^3 v \rightarrow 4\pi v^2 dv$
- and with the incident (E_i), and most probable energy (E₀) of WIMPs:

$$E_{i} = \frac{1}{2}m_{\chi}v^{2}$$
 and $E_{0} = \frac{1}{2}m_{\chi}v_{0}^{2}$

• we obtain for the differential rate:

$$\frac{dR}{dE_R}(E_R) = \int_{E_R/r}^{\infty} \frac{dR(E_i)}{E_i r} = \frac{R_0}{r\left(\frac{1}{2}m_{\chi}v_0^2\right)^2} \int_{v_{\min}}^{\infty} e^{-\frac{(\bar{v})^2}{v_0^2}} v dv = \frac{R_0}{E_0 r} e^{-\frac{E_R}{E_0 r}}$$
$$v_{\min} = \sqrt{\frac{2E_R}{r \cdot m_{\chi}}}$$
simplified expression which we introduced at the beginning

1. Correction: galactic escape velocity vesc

• For a finite escape velocity v_{esc} (and still $v_E = 0$)

$$\left| \vec{v} + \vec{v}_E \right| = v_{esc}$$
 WIMP velocity in the galaxy frame

• we obtain for the differential rate:

$$\frac{dR}{dE_R} = \frac{k_0}{k_1(v_{esc}, 0)} \frac{R_0}{E_0 r} \left(e^{-\frac{E_R}{E_0 r}} - e^{-\frac{(v_{esc})^2}{v_0^2}} \right)$$

• **Example:** if we use the value $v_{esc} \sim 600 \text{ km/s}^*$, and $v_0 = 220 \text{ km/s}$, we obtain:

$$\frac{k_0}{k_1} = 0.9965 \qquad \frac{R(0, v_{esc})}{R_0} = 0.9948 \qquad ^* v_{esc} = 462 - 640 \text{ km/s with } 90\% \text{ CL} \\ \text{(data from RAVE survey, MNRAS 379, 2007)} \end{cases}$$

• and for $m_{\chi} = m_N = 100 \text{ GeV} => maximum E_R = 200 \text{ keV}$

 \blacksquare cutoff energy >> mean recoil energy <E_R> \approx 30 keV

2. Correction: velocity of the Earth v_E

- Clearly the Earth is moving, thus $\mathbf{v}_{E} \neq 0$, and $v_{E} \sim v_{0} \approx 220$ km/s
- A complete calculation yields (see Appendix in Ref [5]):

$$\frac{dR}{dE_R} = \frac{k_0}{k_1} \frac{R_0}{E_0 r} \left\{ \frac{\sqrt{\pi} v_0}{4v_E} \left[erf\left(\frac{v_{\min} + v_E}{v_0}\right) - erf\left(\frac{v_{\min} - v_E}{v_0}\right) \right] - e^{-\frac{(v_{esc})^2}{v_0^2}} \right\}$$

• with the error function being defines as:

$$erf(\mathbf{x}) = \frac{2}{\sqrt{\pi}} \int_{0}^{\mathbf{x}} e^{-t^{2}} dt$$
$$v_{\min} = v_{0} \sqrt{\frac{E_{R}}{E_{0}r}}$$
$$k_{1} = k_{0} \left[erf\left(\frac{v_{esc}}{v_{0}}\right) - \frac{2}{\sqrt{\pi}} \frac{v_{esc}}{v_{0}} e^{-\frac{(v_{esc})^{2}}{v_{0}^{2}}} \right]$$

)

• and:

Signal Modulation: Annual Effect

 The velocity of the Earth varies over the year as the Earth moves around the Sun, and can be written as [in km/s]:



Signal Modulation: Recoil Direction

The motion of the detector with respected to the Galactic rest frame produces a directional signal: the WIMP flux in the laboratory frame is sharply peaked in the direction of motion of the Earth; this results in a recoil spectrum which is also peaked in this direction.



The differential angular spectrum is given by:

$$\frac{d^2 R}{dE_R d(\cos\theta)} = \frac{1}{2} \frac{R_0}{E_0 r} e^{-\frac{\left(v_E \cos\theta - v_{\min}\right)^2}{v_0^2}}$$

 \Rightarrow asymmetry: more events in forward than in backward direction (about factor 10 difference)

→ few 10s-100s events need to distinguish between halo models [depending on the E_{th} of the detector and whether the detector can measure the sense of the recoil]

Additional "Corrections" to the Differential Rate

$$\frac{dR}{dE_R} = R_0 S(E_R) F^2(E_R) I$$

- so far we have discussed the **spectral function S(E**_R)
- it contains the kinematic of the scattering, and the time dependence of the signal
- we now discuss
 - ightarrow **F**²(**E**_R): form factor corrections, with E_R = q²/2m_X
 - ➡ I: type of interaction
- in general, for NR particles (v << c) the scalar and axial vector interactions dominate; we will thus
 consider spin-independent and spin-dependent couplings

Nuclear form factor and spin-independent couplings

- Scattering amplitude: Born approximation $\vec{q} = \hbar \left(\vec{k}' \vec{k} \right)$
- Spin-independent scattering is coherent $\ \ \lambda = \hbar/q \sim \$ few fm

• with r_n = nuclear radius, $r_n \approx 1.2 \text{ A}^{1/3}$ fm, s = 1fm (skin thickness)

Nuclear form factor and spin-independent couplings

• Loss of coherence as larger momentum transfers probes smaller scales:



Spin independent cross section

• The differential cross section can be written as:

$$\frac{d\sigma(q)}{dq^2} = \frac{\sigma_0 F^2(q)}{4\mu^2 v^2} \longrightarrow re$$

elative velocity in centerof-mass frame

- where σ_0 = total cross section for F(q) = 1
- From Fermi's Golden Rule it follows:

$$\frac{d\sigma(q)}{dq^2} = \frac{1}{\pi v^2} |M|^2 = \frac{1}{\pi v^2} f_n^2 A^2 F^2(q)$$

• We can then identify the total cross section σ₀ for F(q)=1 as:



Spin independent cross section and differential rate

• Putting now everything together:

$$\frac{d\sigma(q)}{dq^2} = \frac{1}{4m_n^2 v^2} \sigma_n A^2 F^2(q) \qquad \text{differential cross section}$$

$$\frac{dR}{dE_{R}} = \frac{R_{0}}{E_{0}r} e^{-\frac{E_{R}}{E_{0}r}} F^{2}(q)$$

differential recoil energy spectrum



WIMP Mass and SI Cross Section

• Predictions from supersymmetry $[10^{-8} \text{ pb} = 10^{-44} \text{ cm}^2]$:



Spin independent cross section and differential rate

• Expected rates for different detector materials



Nuclear form factor and spin dependent couplings

 For spin dependent couplings the scattering amplitude is dominated by the unpaired nucleon: the coupling is to the total nuclear spin J (paired nucleons 1 tend to cancel):

$$\frac{d\sigma(\boldsymbol{q})}{d\boldsymbol{q}^2} = \frac{8}{\pi v^2} \Lambda^2 \boldsymbol{G}_F^2 \boldsymbol{J} (\boldsymbol{J}+1) \boldsymbol{F}^2(\boldsymbol{q})$$

- with: G_F = Fermi constant, J = nuclear spin, $F^2(q)$ = form factor for spin dependent interactions
- and

$$\Lambda = \frac{1}{J} \left[a_p \left\langle S_p \right\rangle + a_n \left\langle S_n \right\rangle \right]$$

- a_p , a_n : effective coupling of the WIMPs to protons and neutrons, typically α/m_W^2
- and the expectation values of the proton and neutron spins in the nucleus

$$S_{p,n}
angle = \left\langle N \left| S_{p,n} \right| N \right\rangle$$
 measure the amount of spin carried by the p- and n-groups inside the nucleus

Nuclear form factor and spin dependent couplings

• Form factor example: simplified, based on model with valence nucleons in a thin shell:

$$F(qr_n) = j_0(qr_n) = \frac{\sin(qr_n)}{qr_n}$$

 Better: detailed calculations based on realistic nuclear models
 -for instance, the conventional nuclear shell model using reasonable nuclear

Hamiltonians

- cross check by agreement of predicted versus measured magnetic moment of the nucleus (since the matrix element for χN scattering is similar to the magnetic moment operator)



WIMP Mass and SD Cross Section

• Predictions from supersymmetry $[10^{-8} \text{ pb} = 10^{-44} \text{ cm}^2]$:





Summary: Signal Characteristics of a WIMP

- A² dependence of rates
- coherence loss (for $q \sim \mu v \sim 1/r_n \sim 200 \text{ MeV}$)
- relative rates, for instance in Ge/Si, Ar/Xe,...
- dependance on WIMP mass
- time dependence of the signal (annual, diurnal)





Detection of WIMPs: Signal and Backgrounds



Quenching Factor and Discrimination

- WIMPs (and neutrons) scatter off nuclei
- Most background noise sources (gammas, electrons) scatter off electrons
- Detectors have a different response to nuclear recoils than to electron recoils
- Quenching factor (QF) = describes the difference in the amount of visible energy in a detector for these two classes of events
 - keVee = measured signal from an electron recoil
 - ➡ keVr = measured signal from a nuclear recoil

• For nuclear recoil events:

$$E_{visible}(keVee) = QF \times E_{recoil}(keVr)$$

 The two energy scales are calibrated with gamma (⁵⁷Co, ¹³³Ba, ¹³⁷Cs, ⁶⁰Co, etc) and neutron (AmBe, ²⁵²Cf, n-generator, etc) sources

Quenching Factor and Discrimination

• The quenching factor allows to distinguish between electron and nuclear recoils if two simultaneous detection mechanisms are used

• Example:

- charge and phonons in Ge
- E_{visible} ~ 1/3 E_{recoil} for nuclear recoils
 - ➡ QF ~ 30% in Ge
- ER = background
- NR = WIMPs (or neutron backgrounds)



- Radioactivity of surroundings
- Radioactivity of detector and shield materials
- Cosmic rays and secondary reactions
- Remember: activity of a source
- Do you know?

$$A = \frac{dN}{dt} = -\lambda N$$

N = number of radioactive nuclei λ = decay constant, T_{1/2} = ln2/ λ =ln2 T [A] = Bq = 1 decay/s (1Ci = 3.7 x 10¹⁰ decays/s = A [1g pure ²²⁶Ra])

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1. how much radioactivity (in Bq) is in your body? where from?

2. how many radon atoms escape per 1 m² of ground, per s?

3. how many plutonium atoms you find in 1 kg of soil?

- Radioactivity of surroundings
- Radioactivity of detector and shield materials
- Cosmic rays and secondary reactions
- Remember: activity of a source
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1. how much radioactivity (in Bq) is in your body? where from?

1. 4000 Bq from ¹⁴C, 4000 Bq from ⁴⁰K (e^{-} + 400 1.4 MeV γ + 8000 v_{e})

- 2. how many radon atoms escape per 1 m² of ground, per s?
- 2. 7000 atoms/m² s
- 3. how many plutonium atoms you find in 1 kg of soil?
- 3. 10 millions (transmutation of ²³⁸U by fast CR neutrons), soil: 1 3 mg U per kg

- External, natural radioactivity: ²³⁸U, ²³⁸Th, ⁴⁰K decays in rock and concrete walls of the laboratory => mostly gammas and neutrons from (α,n) and fission reactions
- Radon decays in air:
 - passive shields: Pb against the gammas, polyethylene/water against neutrons
 - active shields: large water Cerenkov detectors or scintillators for gammas and neutrons





- Internal radioactivity: ²³⁸U, ²³⁸Th, ⁴⁰K, ¹³⁷Cs, ⁶⁰Co, ³⁹Ar, ⁸⁵Kr, ... decays in the detector materials, target medium and shields
- Ultra-pure Ge spectrometers (as well as other methods) are used to screen the materials before using them in a detector, down to parts-per-billion (ppb) (or lower) levels



- Cosmic rays and secondary/tertiary particles: go underground!
- Hadronic component (n, p): reduced by few meter water equivalent (mwe)



Flux of cosmic ray secondaries and tertiary-produced neutrons in a typical Pb shield vs shielding depth Gerd Heusser, 1995



- Most problematic: muons and muon induced neutrons
 - ⇒go deep underground, several laboratories, worldwide



Site (multiple levels given in ft)	Relative muon flux	Relative neutron flux T > 10 MeV
WIPP (2130 ft) (1500 mwe)	× 65	× 45
Soudan (2070 mwe)	$\times 30$	× 25
Kamioke	× 12	×11
Boulby	$\times 4$	$\times 4$
Gran Sasso (3700 mwe)		
Frejus (4000 mwe)	$\times 1$	$\times 1$
Homestake (4860 ft)		
Mont Blanc	$\times 6^{-1}$	$\times 6^{-1}$
Sudbury	$\times 25^{-1}$	$\times 25^{-1}$
Homestake (8200 ft)	$\times 50^{-1}$	$\times 50^{-1}$

compiled by: R. Gaitskell

- Activation of detector and other materials during production and transportation at the Earth's surface. A precise calculation requires:
 - cosmic ray spectrum (varies with geomagnetic latitude)
 - cross section for the production of isotopes (only few are directly measured)
- production is dominated by (n,x) reactions (95%) and (p,x) reactions (5%)

	Isotope	Decay	Half life	Energy in Ge [keV]	Activity [µBq/kg]
production in Ge after 30d exposure at the Earth's surface and 1 yr storage below ground	зН	β-	12.33 yr	E _{max(β-)} =18.6	2
	49 V	EC	330 d	E _{K(Ti)} = 5	1.6
	⁵⁴ Mn	EC, β+	312 d	$E_{K(Cr)} = 5.4, E_{Y} = 841$	0.95
	⁵⁵ Fe	EC	2.7 yr	E _{K(Mn)} = 6	0.66
	⁵⁷ Co	EC	272 d	E _{K(Fe)} =6.4, E _Y =128	1.3
	⁶⁰ Co	β-	5.3 yr	$E_{max(\beta-)}=318, E_{\gamma}=1173, 1333$	0.2
	⁶³ Ni	β-	100 yr	E _{max(β-)} =67	0.009
	⁶⁵ Zn	EC, β+	244 d	$E^{K(Cu)} = 9, E_{Y} = 1125$	9.2
	⁶⁸ Ge	EC	271 d	E _{K(Ga)} = 10.4	172

Neutron Backgrounds

- MeV neutrons can mimic WIMPs by elastically scattering from the target nuclei
- the rates of neutrons from detector materials and rock are calculated taking into account the exact material composition, the α energies and cross sections for (α,n) and fission reactions and the measured U/Th contents



Neutrons: how can we distinguish them from WIMPs?

- ➡ mean free path of few cm (neutrons) versus 10¹⁰ m (WIMP)
- material dependence of differential recoil spectrum
- → time dependence of WIMP signal (if neutron background is measured to be constant in time)



Detector strategies

Aggressively reduce the absolute background	Background reduction by pulse shape analysis and/or self-shielding	Background rejection based on simultaneous detection of two signals	Other detector strategies
State of the art: (primary goal is 0vββ decay): Heidelberg-Moscow HDMS IGEX Near future projects: GERDA MAJORANA	Large mass, simple detectors: Nal (DAMA, LIBRA, ANAIS, NAIAD) CsI (KIMS) Large liquid noble gas detectors: XMASS, CLEAN, DEAP	Charge/phonon (CDMS, EDELWEISS, SuperCDMS, EURECA) Light/phonon (CRESST, ROSEBUD, EURECA) Charge/light (XENON, ZEPLIN, LUX, ArDM, WARP, DARWIN)	Large bubble chambers - insensitive to electromagnetic background (COUPP, PICASSO) Low-pressure gas detectors, sensitive to the direction of the nuclear recoil (DRIFT, DMTPC, NEWAGE)

In addition:

- → reject multiple scattered events and events close to detector boundaries
- \rightarrow look for an annual and a diurnal modulation in the event rate

End