# An introduction to cosmology

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Aim:

- What is the universe made of ?
- How do we know ?

Never underestimate the pleasure people have when they hear something they already know

E. Fermi

## Chapter One

## The Cosmological Principle and Basic Equations

## The Cosmological Principle

States that, on large scales, the universe is homogeneous and isotropic, an old and bold idea that goes back to Copernic.

The cosmological principle is also sometime called the mediocrity principle...

**Based on Three Observational Facts** 

- 1. The large scale distribution of matter
- 2. The isotropy of cosmological signals, most notably the cosmic microwave background radiation
- 3. The recession of distant galaxies

## Fact # 1. The large scale distribution of matter

- N.B.: Light-year  $1 \lg \approx 10^{16} m$ 
  - Parsec  $1pc \approx 3.3ly$

The universe is clumpy at first sight.

there are planets, stars, galaxies, clusters of galaxies,...

However the average mass density  $\rho$  on a given scale decreases as the scale increases.

Furthermore, the largest structures seen are clusters of clusters or superclusters, on scale  $\sim 100\,Mpc.$ 

Beyond this, the universe is essentially homogeneous (no more structure seen and  $\rho \rightarrow$  constant).

This is seen in large scales galaxy surveys. This one is from the 2dF collaboration (http://www.mso.anu.edu.au/2dFGRS/), a survey of the position of 382,323 galaxies, up to a distance of  $\sim$  1 Gpc.



This figure provides more quantitative informations (Peacock and Dodds 1994, MNRAS, 267, 1020).

It shows you the matter density contrast  $\Delta = \delta \rho / \rho$ , where  $\rho$  is the large scale average mass density, as a function of the radius of some averaging fonction (top hat). The trend is clear.



### Fact # 2. Isotropy of cosmological signals

Signals that are extra-galactic origin show remarkable isotropy.

This is an observation of the distribution of galaxies on the sky (the APM galaxy survey, about  $210^6$  galaxies). The QSO (quasars,...) distribution is similar.



Distribution of Gamma Ray Bursts



The most remarkable is the nearly perfectly isotropic Cosmic Microwave Background Radiation (CMBR) with an almost perfect **black body** spectrum, with temperature  $T = 2.725 \pm 0.002K$  (peak frequency 160 GHz, wavelength 1.9 mm). Discovered in 1965 by Penzias and Wilson but existence anticipated by Gamow as early as 1948.



This spectrum was obtained by the COBE satellite (FIRAS, 1994) together with a black body curve at T=2.725 K. The error flags have been enlarged by a factor of 400 so that you can actually see them.

First departure from isotropy starts with a dipole, with  $\Delta T \sim 3.3$  mK. This is signature of the Sun's motion with respect to the reference frame of the CMB. The inferred velocity is  $\nu_\odot \approx 370$  km/s, corresponding to  $\beta = \nu/c \sim 10^{-3}$ .

The dipole effect has been first observed in 1969 (Charles H. Lineweaver, astroph/9609034 for a review).

True anisotropies arise at a much smaller scale,  $\Delta T/T \sim 10^{-5}$ . We will discus CMB anisotropies in the 4t lecture.



### By the way

Isotropy and homogeneity are independent concepts.

However isotropy around any two points implies homogeneity.



### Fact # 3. Recession of distant galaxies

What Hubble did:

He estimated the distance d'' to far away galaxies using cepheids (a class of variable stars). Determining distances in cosmology was, and still is, a tricky business.

He also measured their spectrum (that was, and still is, relatively easy)

He observed systematic deviations between emitted and observed wavelengths

$$rac{\lambda_{
m o}-\lambda_{e}}{\lambda_{e}}=z \hspace{1cm} z={
m Redshift}$$

and interpreted this as a Doppler effect

$$\frac{\lambda_{\rm o}-\lambda_e}{\lambda_e}=z\approx\frac{\nu}{\rm c}$$



The historical Hubble diagram (1929).

## He got (1929)

 $v = H_0 d$  Hubble's law

with

$$H_0 \approx 500 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$$
 Hubble's constant



 $H_0=64 km\ s^{-1} Mpc^{-1}$ , which is closed to the currently prefered value. Here we define  $H_0$  = h100km  $s^{-1} Mpc^{-1}$  and adopt h  $\approx$  0.7  $\approx$  2/3,  $h^2\approx 1/2.$ 

For a list of published values of the Hubble constant see

http://www.cfa.harvard.edu/ huchra/hubble.plot.dat

Simplicio:

"The galaxies are flying away from us. We are at the center of the Universe!" Sagredo:

"Or it means that the universe is uniform and galaxies are moving away from each others..."

Salviati:

"Right, but since Einstein we say that the universe is expanding!"



You may check that the Hubble law is the only solution consistent with an homogoneous and isotropic universe (ie  $v \propto d^2$  does not work).

Conversely, we make take the Hubble law as a further argument supporting the cosmological principle.

## Hubble and the age of the universe

The Hubble constant has the dimension of 1/time. In particular

$$I/H_0 = h^{-1}9.78 \cdot 10^9 \, \text{yr}$$

If we use the present value,  $h\approx 2/3,$  we get a timescale of  $15\cdot 10^9$  years, older than the oldest known objects (globular clusters, with  $t_{GC}\sim 12\cdot 10^9$  years.

In 1929, the timescale was closer to  $2 \cdot 10^9$  years, younger than the age of the solar system.

If v was constant (on a given wordline),  $H^{-1}$  would indeed be the "age of the universe", *i.e.* the time elapsed since the beginning of expansion:

$$v = const = \dot{a}(t)x \rightarrow a(t) \propto t \rightarrow H(t) = \dot{a}/a = 1/t$$

Since gravity is attractive, the galaxies should slow down.

We should expect that the universe is younger than  $1/H_0$ .

## **Basic equations**

### A. Kinematics

We take space (slicing) as homogeneous & isotropic.

Then there is a universal t time coordinate ("the age of the universe" = time for observers that see space slices as homogeneous & isotropic)

Galaxies are taken to be at rest (no peculiar velocity to first approximation) with respect to comoving coordinates  $(\underline{\chi}, \theta, \phi)$ .

Their collective motion is due to the expansion of space. It is encompassed by the scale factor a(t). We take  $a_{today} = a_0 = 1$ .



#### Physical distance $d_P$

Actual distance from to *e.g.* a galaxy at a given moment of time. (For other distances, see later.)

 $d_P(t) = \mathfrak{a}(t)\chi$ 

Since we took  $a_0 = 1$ , comoving distances are physical distances today.

Recession velocity:

$$\begin{aligned} \nu &= \dot{d}_{P} = \dot{a}\chi = \frac{\dot{a}}{a}d_{P} \\ &\equiv Hd_{P} \end{aligned}$$

This relation is called the Hubble law and it is exact (*ie* not an approximation).

Exercise 1.

Show that the Hubble law is consistent with spatial isotropy and homogeneity.

Exercise 2.

How could you measure the distance  $d_P$ ?

Exercise 3.

What is the velocity of an object at cosmological distance  $d_{P}=1/H?\;$  Is this a problem?

#### The Robertson-Walker metrics

Geometrically space is described by its metric (infinitesimal) distance between two points.

Homogeneous & isotropic space : only three such geometries (but many more topologies) in any dimensions.

Sphere, plane and hyperbolic plane = surfaces of constant curvature  $K \propto 1/R^2$ .

$$d\vec{x}^2 = d\chi^2 + S_{K}^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)$$

Flat space:  $K = 0; R \to \infty$ 



Spherical space: 
$$K > 0 = 1/R^2$$









Hyperbolic space:  $K < 0 = -1/R^2$ 







 $\alpha + \beta + \gamma < \pi$ 

Finally, a spacetime interval in an expanding universe is given by (Robertson & Walker):

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathfrak{a}(t)^2 \mathrm{d}\vec{x}^2$$

As a simple application, consider a photon of wavelength  $\lambda_e$  emitted by galaxy A and observed by galaxy O with wavelength  $\lambda_o$ . We can show that  $\lambda \propto a(t)$ .

The comoving distance between O and A is  $\chi = \text{constant}$ . Photons travel on light-cones,  $ds^2 = 0$  or

$$d\chi^2 = dt^2/\alpha(t)^2$$

Thus

$$x_{galaxy} = \int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{t_e+T_e}^{t_o+T_o} \frac{dt}{a(t)}$$

where the period  $T=\lambda$  (remember  $c=1). You can use <math display="inline">T_{e,o}\ll t_{e,o}$  to show that (exercise)

$$\frac{\lambda_o}{\lambda_e} = \frac{a(t_o)}{a(t_e)}$$

#### **Redshift revisited**

The ratio

$$\frac{\lambda_{\rm o}-\lambda_e}{\lambda_e}=z$$

is what we called the *redshift factor z*.

If we normalize  $a(t_0) = 1$ , we thus have

$$a(t_e) = \frac{1}{1+z}$$

The redshift is a measure of the scale of the universe at the time of emission.

#### B. Dynamics

The universe is considered to be filled by an homogeneous & isotropic fluid = ideal fluid.

An ideal fluid is described by

### $\rho(t)$ its energy density and p(t) its pressure

Depending on the context, the cosmic fluid is made of elementary particles, massive (non-relativistic = NR) or massless (or relativistic, or radiation = R) or whole galaxies (treated as point-like objects). More strange fluids may be necessary (i.e. a cosmological constant)

For instance, for a NR objects:  $\rho \approx mc^2 n$  with n the density and  $p \approx 0$ .

Consider a spherical region of the universe, of mass  $M=4/3\pi\rho d^3,$  an a test galaxy of mass m.



If you forget about the rest of the universe, the force felt by the test galaxy is

$$m\ddot{d} = -\frac{GMm}{d^2} = -\frac{4\pi G}{3c^2}m\rho d$$

where G is Newton's constant.

(In high energy units  $G = 1/M_{Planck}^2$  with  $M_{Planck} = 1.2 \cdot 10^{19} \text{ GeV}$ )

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Drop the m (principle of equivalence), drop the  $\chi$  in  $d=a(t)\chi.$  You get The Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\rho$$

This is the correct equation for non-relativistic matter.

If pressure ( = kinetic energy density) is important, then

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} \ (\rho + 3p)$$

The effective gravitational mass-energy is thus  $\rho + 3p^*$ .

Pressure, if positive, is as attractive as matter.

\*This is a relativistic effect, so you need the Einstein equations to derive this result.
## Important:

If  $\rho+3p > 0$  (normal matter or radiation) the expansion of the universe is decelerating (slowing down).

fluid energy conservation:

The expansion of FLRW universe is adiabatic (no entropy creation). Then an element of fluid of volume V must satisfy

$$dE \equiv \rho dV + V d\rho = -p dV \qquad \text{with} \qquad V \propto a^3$$

Then

$$\dot{\rho}=-3H(\rho+p)$$

Three extreme types of fluid are usually envisioned:

Dust, non-relativistic matter:

$$p = 0 \longrightarrow \rho \propto a^{-3}$$

Radiation, relativistic matter:

$$p = \frac{\rho}{3} \longrightarrow \rho \propto a^{-4}$$

Cosmological constant, dark energy:

$$p = -\rho \longrightarrow \rho = \text{const.}$$

# The Friedmann equation

Combining the Raychaudhuri equation and energy conservation, you get (up to an integration constant, exercise) the most important equation of cosmology (I set c = 1)

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

To match the integration constant with the curvature K , you unfortunately need General Relativity.

### C. Cosmological Solutions

What is Friedmann good for?

1. The Friedmann equation relates three important observable quantities: the Hubble parameter H, the total energy density  $\rho$  and the curvature K.

Dividing the LHS and the RHS of Friedmann by  $\mathrm{H}^2$  gives the most important equation of cosmology ;-)

$$1 = \Omega - \frac{K}{a^2 H^2}$$

where

$$\Omega = \frac{\rho}{\rho_c}$$

with  $\rho_c$  is the **critical density** defined as

$$\rho_{c} = \frac{3H^{2}}{8\pi G}$$

Today

$$\rho_{c0} = \frac{3H_0^2}{8\pi G} = 1.88 \,h^2 \times 10^{-29} \,g \cdot cm^{-3}$$
  
= 1.1 h^2 10 GeV \cdot m^{-3}  
= 2.775 h^2 10^{11} \,M\_{\odot} \cdot Mpc^{-3}  
= (3 \times 10^{-3} \,eV)^4 \,h^2

A universe with  $\Omega > 1$  has a spherical geometry,  $\Omega < 1$  is hyperbolic while  $\Omega = 1$  is flat. Note that only the latter has  $\Omega = \text{constant}$  (more on this in lecture 4).

2. Multiplying Friedmann by  $a^2$  and dividing by 2 gives the most important equation of cosmology (no kidding)



 $K=\ensuremath{0}$  solutions, also valid for Early Universe

Ma	atter			
	$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$	$H = \frac{2}{3t}$	$D_H = \frac{1}{2}$	2 H
Radiation				
	$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$	$H = \frac{1}{2t}$	$D_H = \frac{1}{1}$	1 H
$\underline{\Lambda}$				
	$a(t) = a(t_i) \exp(H(t{-}t_i))$	H = cor	nst l	$D_{H} \rightarrow \infty$

MATTER + COSMOLOGICAL CONSTRANT



The solution  $a(t) \propto t$  corresponding to an empty, non-decelerating, universe gives a very simple expression for the age of the universe (time lapsed since a = 0)

$$t_0 = \frac{1}{H_0} \approx 15 \cdot 10^9 \, \text{yr}$$

A cosmic fluid may decelerate (attractive matter) or accelerate ( $\Lambda$ ) the expansion, giving respectively a younger or older universe. For instance

$$a \propto t^{2/3} \rightarrow H = \frac{2}{3t} \rightarrow t_0 = \frac{2}{3H_0} \approx 10 \cdot 10^9 \text{ yr}$$

for a flat matter dominated universe.

This is less than the age of globular clusters. The universe can not be flat, matter dominated.

Look-back time for some relevant a(t) solutions



From bottom to top: flat matter dominated (blue), empty (black), 30 % matter + 70 %  $\Lambda$  (red), 10 % matter + 90 %  $\Lambda$  (yellow)

# The effect of a cosmological constant (flat universe)



A cosmological constant has been used ever since its introduction by Einstein to solve an "age crisis" of the universe

Curvature has also an impact.



Cosmological calculator:

http://www.astro.ucla.edu/ wright/CosmoCalc.html

# Chapter two

Mapping the Cosmological espansion and the Early Universe

# Mapping the cosmological expansion

#### Luminous distance

<u>Definition</u>: the **absolute luminosity** L of an object is the energy it radiates per unit of time. This energy could be in a range of frequencies or integrated over the whole spectrum of radiation. We consider only the latter here. Its **apparent luminosity** l is the energy received per unit area per unit of time.

Suppose you know the absolute luminosity L of a distant object (galaxy, quasar,...) and measure its apparent luminosity F equal to the flux of energy per steradian. In Euclidean space

$$\mathsf{F} = \frac{\mathsf{L}}{4\pi \mathsf{d}_{\mathsf{L}}^2}$$

We can thus define the luminous distance by

$$d_{L} = \left(\frac{L}{4\pi F}\right)^{\frac{1}{2}}$$

This definition applies to an expanding universe, provided we understand how the apparent luminosity is affected by expansion.

Using comoving distance, in a flat, static universe the flux observed today is

$$F = \frac{L}{4\pi\chi^2}$$

What about a curved, expanding universe?

In <u>curved space</u> we must simply replace  $\chi$  by  $S_k(\chi)$ . Furthermore, because of expansion the **energy** of a photon is redshifted by a factor of a = 1/(1+z) between emission and reception. Last, we must take into account the fact that the **rate** of photon reception is smaller than the rate of emission by a factor of a = 1/(1+z).

Altogether, the flux of energy received is

$$\mathsf{F} = \frac{\mathsf{La}^2}{4\pi \mathsf{S}^2_\mathsf{k}(\chi)}$$

This motivates us to define the luminous distance as

$$d_L = \frac{S_k(\chi)}{a} = (1+z)S_k(\chi)$$

Note that, in a flat universe, this is a factor of 1/a larger than the physical distance today

$$d_{L} = \frac{d_{P}}{a}$$

Objects look fainter, *i.e.* further away, because of expansion.



#### Angular diameter distance

As an alternative definition of distance, suppose we know the physical size D of an object. In Euclidean space, the apparent diameter  $\delta$  is

$$\delta = \frac{D}{d_A}$$

if the object is at distance  $d_A$ . Thus we define

$$d_A = \frac{D}{\delta}$$

and asked how D and  $\delta$  are affected in an expanding universe.

According to the FRW metric, the angle sustained by an object of physical size D at comoving distance  $\boldsymbol{\chi}$  is

$$\delta = \frac{D}{\alpha S_k(\chi)}$$

which gives

$$d_A = \alpha S_k(\chi) = \frac{S_k(\chi)}{1+z}$$

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Note that in a flat universe, the angular distance is  $\underline{smaller}$  than the physical distance by a factor of  $\mathfrak{a},$ 

 $d_A = \mathfrak{a} d_P$ 

### Horizon

The angular diameter distance has interesting properties, related to the existence of a particle **horizon**.

Consider for instance a flat matter dominated universe. The comoving distance to a an object at redshift  $\chi$  is

$$\chi = \int_{t(a)}^{t_0} \frac{dt}{a(t)} = \int_a^1 \frac{da}{a^2 H} = \frac{2}{H_0} \left[ 1 - \frac{1}{\sqrt{1+z}} \right]$$

The comoving distance is  $\chi = z/H_0$  for small z but then asymptots to  $2/H_0$  for large z, which is the size of the particle horizon today = largest distance that could have been travelled by a particle moving at the speed of light. (More on this in the 4th lecture).

Consequently the angular distance increases at small redshift, as expected, but decrease around  $z \sim 1$ : the apparent diameter of a category of objects of fixed physical size increases with distance!

The effect of curvature is also important. In a curved universe,

$$d_{A}(z) = \frac{1}{(1+z)H_{0}\sqrt{|\Omega_{k}|}} \begin{cases} \sinh(\sqrt{\Omega_{k}}H_{0}\chi) & \Omega_{k} > 1\\ \sin(\sqrt{-\Omega_{k}}H_{0}\chi) & \Omega_{k} < 1 \end{cases}$$
(1)

where  $\Omega_k = 1 - \Omega = -K/H_0^2$ . An object of fixed size, at fixed comoving distance, appears larger in a closed universe than in a flat universe. The converse holds in an open universe.

#### The Hubble law and the luminosity distance-redshift relation

We saw that luminous, physical and angular distance are equal at small z. In this section we extend the Hubble law to second order in z for the luminous distance.

To express  $d_L$  as a function of the redshift z we need to eliminate the reference to  $\chi$ . This is easy if we can solve the Friedmann equation for a(t), as we did in the figure in the preceding section.

Otherwise, if we consider small z, we can derive an approximate relation by expanding a(t) near  $t = t_0$ . This approach does not rest on theoretical prejudices (*ie* the validity of Einstein equations) but requires the introduction of a priori arbitrary parameters. We consider the expansion to second order in z only for convinience.

We need  $\chi$  as a function of time t. From the geodesic motion of a photon, we have

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \chi \tag{2}$$

Expand a(t) in the vicinity of  $t_0$ :

$$a(t) = a(t_0) + \dot{a}(t_0)(t - t_0) + \frac{1}{2}\ddot{a}(t_0)(t - t_0)^2 + \dots$$
  
= 1 + H\_0(t - t\_0) -  $\frac{1}{2}q_0H_0^2(t - t_0)^2 + \dots$  (3)

(4)

where  $H_0 = \dot{a}/a$  today and  $q_0 = -\ddot{a}_0/H_0$  is called the **deceleration parameter**.

Inserting the expansion of a(t) in the *lhs* of Eq(2), we get

$$\chi = (t_0 - t_1) + \frac{1}{2}H_0(t_0 - t_1)^2 + \dots$$

In a static universe, it takes a time  $t_0-t_1$  for light to travel a physical distance  $\chi$  (remember comoving distance = physical distance today). This takes less time in an expanding universe since the source was closer,  $a(t_1)\chi < \chi$ .

Finally we need to relate  $t_0 - t_1$  and z. As

$$1 + z = \frac{a_0}{a_1} = \frac{1}{a(t_1)}$$

we have

$$z = H_0(t_0 - t_1) + \left(1 + \frac{q_0}{2}\right) H_0^2(t_0 - t_1)^2 + \dots$$

or

$$t_0 - t_1 = H_0^{-1} \left( z - (1 + q_0/2) z^2 + \ldots \right)$$

Now, if we limit the expansion to second order in z, you can verify that the correction due to spatial curvature is to next order. That is, we can take

 $\chi\approx S_k(\chi)$ 

Putting everything together, we find that the luminous distance to the source is related to redshift by

$$d_L H_0 = z + \frac{1}{2}(1 - q_0)^2 z^2 + \dots$$

Note that there is deviation from the Hubble law even in a universe with vanishing deceleration (Milne universe). This is both because of the existence of an horizon and of the diming of light by expansion.

The Hubble diagram below compares three notion of distance we have just discussed in the case of a flat, matter dominated universe. Notice the bending downward of the angular diameter distance.



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The Hubble diagrams below compare how various distances for various cosmological models.



### Magnitude-distance relation

Magnitude is one of these historical units, based on the approximately logarithmic sensibility of the eye, which make astronomy such a delight. The use of parsecs instead of light-years is another instance.

The relation between magnitude and to luminous distance is given by

$$m = M + 5\log_{10}\left(\frac{d_L}{10pc}\right) + K(z)$$

where M is the magnitude of an object as seen from a distance of 10pc. The K-correction takes into account the fact that instruments are sensitive not to the total luminosity but to some range of frequencies and that frequencies are shifting because of expansion.

### Cosmological Constant

From observations distant type Ia Supernovae. Here the historical data.



From observations distant type Ia Supernovae. Here a compilation by Ned Wright (http://www.astro.ucla.edu/ wright/cosmolog.htm)



Consider a universe with matter and a cosmological constant.

Using the Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{1}{2}H^2\left(\frac{\rho_M}{\rho_c} + \frac{\rho_\Lambda}{\rho_c} - 3\frac{\rho_\Lambda}{\rho_c}\right)$$

we get for the deceleration parameter today

$$q_{0} = -\frac{\ddot{a}}{aH^{2}}\Big|_{t_{0}} = \frac{1}{2}\left(\Omega_{M} - 2\Omega_{\Lambda}\right)$$



# MATTER / ENERGY in the UNIVERSE



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## The early universe

The CMBR has a spectrum of a black body with temperature T  $\approx$  2.72 K. The energy density of this gas of photons is given by

$$\rho = 2 \int \frac{dk^3}{(2\pi)^3} \frac{\omega(k)}{\exp(\omega(k)/T) - 1} = \frac{\pi^2}{15} T^4$$

with  $\omega(k) = k$  (in high energy units). One may also estimate the number density of photons

$$n = \frac{2\zeta(3)}{\pi^2} T^3 \approx 422 \, \mathrm{cm}^{-3} \, \mathrm{today}$$

and the corresponding fraction of the critical energy density

$$\Omega_{\gamma 0} = 2.48 \ h^{-2} \ 10^{-5}$$

We have seen that  $\rho \propto \alpha^{-4}$  for a gas of relativistic particles. Thus

$$T \propto \frac{1}{a}$$

A remarkable property of the isotropic expansion of the universe is that it preserves the blackbody nature of the photon spectrum. Since both  $\omega \propto 1/a$  and  $T \propto a$ 

$$\frac{1}{\exp(\omega/T) - 1} = \frac{1}{\exp(\omega_0/T_0) - 1}$$

Thus, the existence of the blackbody CMB points unambigously to a phase when

the universe was hot, presumably in thermal equilibrium when light and matter interacted strongly (*ie* frequently).

As T  $\propto a$ , the temperature was very high at early times, eventually higher than the mass of the known (and unknown) particles and the expansion was dominated by radiation.

Let  $g_{\star}$  be the effective number of relativistic degrees of freedom at T

$$\rho = g_{\star} \frac{\pi^2}{30} T^4$$
with

$$g_{\star} = \sum_{i=\text{bosons}} g_i + \frac{7}{8} \sum_{i=\text{fermions}} g_i$$

(the factor of 7/8 comes from Fermi-Dirac vs Bose-Einstein statistics).

Then the expansion rate takes a very simple form in the radiation dominated era (*ie* early universe)

$$H\approx 1.66 g_{\star}^{1/2} \frac{T^2}{M_{\text{pl}}}$$

When was the expansion dominated by radiation?

The universe today contains various fluids

$$\Omega = \Omega_{Matter} + \Omega_{Radiation} + \Omega_{\Lambda} + \dots$$

Then

$$\frac{\Omega_{R}}{\Omega_{M}} = \frac{\Omega_{R,0}}{\Omega_{M,0}}(1+z) \qquad \rightarrow \qquad 1+z_{Eq.} = \frac{\Omega_{M,0}}{\Omega_{R,0}}$$

$$\frac{\Omega_{\Lambda}}{\Omega_{M}} = \frac{\Omega_{\Lambda,0}}{\Omega_{M,0}} \frac{1}{(1+z)^{3}} \qquad \rightarrow \qquad 1+z_{\mathsf{Eq.}} = \left(\frac{\Omega_{\Lambda,0}}{\Omega_{M,0}}\right)^{1/3}$$

Estimates of the energy density of matter gives  $\Omega_{M,0} \approx 0.3$  so, tentatively, we take  $z_{EQ} \approx 10^4$ . Correspondingly

$$T_{EQ} = T_0(1 + z_{EQ}) = 10^4 K \sim 1 eV$$

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For the cosmological constant, we may take  $\Omega_{\Lambda} \approx 0.7$ . Hence

$$\frac{\Omega_{\mathcal{M}}}{\Omega_{\Lambda}} = \frac{\Omega_{\mathcal{M},0}}{\Omega_{\Lambda,0}} (1+z)^3 \approx \frac{0.3}{0.7} (1+z)^3 \approx 1 \quad \text{at equality}$$

implies matter energy and dark energy equality at  $z \approx 0.33$ , corresponding to t  $\approx 10^{10}$  yrs.



# Primordial Nucleosynthesis

The early phase of the universe has presumably left many remnants, one of which is the CMBR.

Another important pillar of the standard model of cosmology is primordial nucleosynthesis of the light elements, *ie* helium ( ${}^{4}He$ ) essentially but also tiny bits of deuterium ( ${}^{2}H$ ) and Lithium  ${}^{7}Li$ .

The temperatures of interest are in the MeV range, characteristics of nuclear processes.

We take the universe to be composed of protons and neutrons, which are non-relativistic at T  $\sim 1 MeV$ , as well as electrons, photons and neutrinos, all relativistic particles.

#### Step 1

In thermodynamic equilibrium, the abundances of neutrons (n) and protons (p) satisfy

$$\frac{n}{p} = e^{-Q/T}$$

with

$$Q = m_n - m_p = 1.293 MeV$$

Hence at high temperatures  $T \gtrsim 1 \text{MeV}$ ,  $n \approx p$ , as expected.

(To get this equation, we need to assume that there are a similar number of neutrinos and antineutrinos. More on this later) Thermo equilibrium is mechanical equilibrium plus chemical equilibrium. The latter is maintained if processes like

$$n + \nu_e \leftrightarrow p + e$$

or

 $n + \bar{e} \leftrightarrow p + \bar{\nu}_e$ 

are occuring "rapidly".

The rate  $\Gamma$  of these processes is controlled by (thermally averaged) weak cross-sections

$$\left<\sigma|\nu|\right>\sim G_F^2 T^2$$

where  $G_F\approx 10^{-5}GeV^{-2}$  (you can guess this purely on dimensional grounds) and the density of target particles  $n\sim T^3$  or

$$\Gamma \sim G_F^2 T^5$$

## What do we mean by "rapid" interactions?

When we consider a specific processus, there are essentially two relevant time scales. The first is the interaction timescale,  $\tau_I = \Gamma^{-1}$ . The other is the age of the universe at the temperature T,  $t \sim H^{-1}$ .

Intuitively, a processus is in equilibrium if  $\tau_I \! \lesssim \! t$  or



Otherwise, it is said to be out-of-equilibrium.

Concretely, the weak processes to which the neutrons and protons take part become inefficient when

$$\Gamma \sim H ~\rightarrow~ G_F^2 T^5 \sim g_\star^{1/2} \frac{T^2}{M_{\text{pl}}}$$

which occurs at (exercise)

$$T_{freeze out} \sim 1 MeV$$

(more precise calculations yield  $T_{FO}\approx 0.8$  MeV). The age of the universe is  $t\approx 1s.$ 

Step 2

At  $T_{FO}$ 

$$\frac{n}{p} = e^{-Q/T_{FO}} \approx 1/5$$

If all the neutrons were to be incorporated in helium nuclei, the mass fraction of  ${}^{4}\text{He}$  (how much baryon mass is in helium) would be

$$X_{\text{He}} = \frac{4n_{\text{He}}}{n+p} = \frac{2n}{n+p} \approx \frac{1}{3}$$

For a first estimate, this is not too bad since the "observed" mass fraction of primordial He is close to 25%.

The explanation for the difference is interesting and instructive.

#### Step 3

Some processes relevant for the formation of <sup>4</sup>He are

$$n+p\leftrightarrow D+\gamma$$

followed by

$$\mathsf{D} + \mathsf{D} \leftrightarrow \mathsf{H} e + \gamma$$

At T  $\sim 1 MeV,$  these processes are fast compared to the expansion rate = thermodynamic equilibrium.

Using the equilibrium abundance of a non-relativistic particle or nuclei of mass  $\mathfrak{m}$  and chemical potential  $\mu$ 

$$n_{Eq} = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$$

and the condition on the chemical potential at equilibrium (chemistry 101)

$$\mu_n + \mu_p = \mu_D$$

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for instance, we can derive the equilibrium mass fraction of Deuterium

 $X_2 \sim \eta e^{B_2/T}$ 

where  $\eta = n_b/n_\gamma$  is the ratio of the baryon to photon densities and  $B_2 = 2.22 MeV$  is the binding energy of deuterium.

For Helium, we would get

$$X_4 \sim \eta^2 e^{B_4/T}$$

with  $B_4 = 28.3 MeV$  while a nucleus made of A nucleons we would have

$$X_A \sim \eta^{A-1} e^{B_A/T}$$

These so-called Saha equations tell us that there is a competition between energy (the exponential factor) and entropy (the number of photons).

The key point is that  $\eta = n_b/n_\gamma$  is a small number. For galaxy abundances only we may estimate that there is less than one nucleon per cubic meter, while there are of the order of  $10^8$  photons per m<sup>3</sup> in the CMB.

From the Saha equations we get that, at  $T_{FO} \sim 1 MeV$ 

$$X_2 \approx 10^{-12}$$
  $X_4 \approx 10^{-23}$   $X_{12} \approx 10^{-108}$ 

Because there are many photons per baryon, nucleosynthesis starts around t  $\approx$  1 minutes (the famous "first three minutes"):



Since the time scale between freeze-out of weak interactions and the real beginning of nucleosynthesis is  $\mathcal{O}(\text{minutes})$ , similar to the half-lifetime of the neutron  $\tau_n \approx 886s$ , the abundance of neutrons changes substantially



$$\frac{n}{p} \approx \frac{1}{7}$$

which gives (most of the neutrons go into Helium nuclei)

 $X_4\approx 0.25$ 

Sonnez trompettes!

The predictions of primordial nucleosynthesis are sensitive indicator for

1. The baryon-to-photon ratio. The larger  $\eta$  the .....  $X_4$  (fill the blanks).

2. The number of degrees of freedom at the time of freeze-out (Peebles 1966). The larger  $g_*$ , the .... H and the .....  $X_4$  (fill the blanks)



1994: observation of primordial deuterium abundance in Lyman- $\alpha$  absorption lines (redshift  $z \approx 2.7$ )



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From the comparison of observations to prediction of primordial nucleosynthesis, we may infer that

$$\eta = (6.0 \pm 0.15) \cdot 10^{-10}$$

or

$$\Omega_{\rm b} {\rm h}^2 = 0.020 \pm 0.005$$

(Particle Data Book)

In lecture 4 we will get another measure of the baryon content of the universe.

In the meantime we may notice that  $\Omega_b\approx 0.04<\Omega_M...$ 

Speaking of neutrinos, primordial nucleosynthesis puts the following limit on the number of light (m  $\lesssim$  MeV) neutrino families

$$1.8 < N_{\nu} < 4.5$$
 (PDG)

## Neutrino families from $e^+e^-$ collisions

#### Number of Light $\nu$ Types from Direct Measurement of Invisible Z Width

In the following, the invisible Z width is obtained from studies of single-photon events from the reaction  $e^+e^- \rightarrow \nu \overline{\nu} \gamma$ . All are obtained from LEP runs in the  $E_{\rm Cm}^{ee}$  range 88–209 GeV.

VALUE	DOCUMENT ID		TECN	COMMENT
2.93±0.05 OUR AVERAGE Error includes scale factor of 1.2.				
$2.98 \!\pm\! 0.05 \!\pm\! 0.04$	ACHARD	04E	L3	1990-2000 LEP runs
$2.86 \pm 0.09$	HEISTER	03C	ALEP	$\sqrt{s}$ =189–209 GeV
$2.69 \pm 0.13 \pm 0.11$	ABBIENDI,G	00D	OPAL	1998 LEP run
$2.84 \pm 0.15 \pm 0.14$	ABREU	00z	DLPH	1997–1998 LEP runs
$2.89 \pm 0.32 \pm 0.19$	ABREU	97J	DLPH	1993–1994 LEP runs
$3.23 \pm 0.16 \pm 0.10$	AKERS	95c	OPAL	1990–1992 LEP runs
$2.68\!\pm\!0.20\!\pm\!0.20$	BUSKULIC	93L	ALEP	1990–1991 LEP runs
ullet $ullet$ $ullet$ We do not use the following data for averages, fits, limits, etc. $ullet$ $ullet$				
$3.01 \pm 0.08$	ACCIARRI	99R	L3	1991–1998 LEP runs
$3.1 \ \pm 0.6 \ \pm 0.1$	ADAM	96C	DLPH	$\sqrt{s}=$ 130, 136 GeV

# MATTER / ENERGY in the UNIVERSE



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# **Chapter Three**

Dark and Ordinary matters

# Dark Matter

Most of matter in the universe is not visible and the evidences that this invisible matter is not made of baryons are accumulating.

There are essentially three evidences. With increasing level of confidence, they are

- 1. The rotation curves of spiral galaxies.
- 2. The dynamics of clusters of galaxies
- 3. The formation of large scale structures (more on this in section 4).

Good reviews are

TASI lectures on Dark Matter, Keith Olive, astro-ph/0301505

Particle dark matter: Evidence, candidates and constraints, by Bertone, Hooper & Silk, hep-ph/0404175

To many, the most convincing evidence for the existence of dark matter is the so-called "Bullet cluster"





X-ray image showing hot baryonic gas (second in mass)



Composite image showing the visible galaxies (least important in mass) and the newtonian potential reconstructed from lensing (most important = dark matter)

Observations concur to indicate that  $\Omega_{dm}\approx$  0.26, about five time the energy density in baryons...

Astrophysical objects, like MACHOS (massive compact halo objects) are essentially excluded.

Many candidates for dark matter are new elementary particles, the most acclaimed being the neutralino the lightest stable supersymmetric particle (LSP).

Another particle physics possibility is the axion, the particle of the field introduced to solve the strong CP problem.

We will limit ourself to the first possibility and will discuss a simple and elegant scenario, based on the existence of massive, weakly interacting particles or WIMPs.

## Hot or Cold dark matter?

We will briefly come back to this issue in the last lecture but let us mention here a distinction between

HOT DARK MATTER (HDM) and COLD DARK MATTER (CDM)

The distinction has to do with their mean free path at the time of matter-radiation equality (first time when structures may start forming).

The latter have negligible mean free path (small momentum) and may form structures on all scales while the former introduce a cut-off in the spectrum of early structures. This is strongly disfavoured by datas.

WIMPs are CDM candidates.

#### The Weakly Interacting Massive Particle Paradigm

Consider a massive, stable, neutral and weakly interacting particle X (assumption 1). The abundance of X is controlled by its annihilation into Standard Model (assumption 2) particles.

$$X + X \leftrightarrow y + z$$

The annihilation rate  $\Gamma$  is given by

$$\Gamma = \langle \sigma | \nu | \rangle n_X$$

where

$$n_X = g_X \left(\frac{m_X T}{2\pi}\right)^{3/2} e^{-m_X/T}$$

Thus we assume that the X has no conserved charge (assumption 3) or at least no charge asymmetry. This could be the case if the X is a Majorana particle, like the LSP typically.

To determine the relic abundance of X particles we should write (and solve) a few Boltzmann equations. Much intuition may be gained by using the thumb rule that equilibrium is maintained as long as

 $L \gtrsim H$ 

where H is the expansion rate.

Thus freeze-out occur at a temperature such that

$$\big\langle \sigma |\nu| \big\rangle n_X \sim g_\star^{1/2} \frac{T_{FO}^2}{M_{\text{pl}}}$$

which gives

$$n_X|_{FO} \sim g_\star^{1/2} \frac{T_{FO}^2}{\langle \sigma |\nu| \rangle M_{\text{pl}}} \qquad \longrightarrow \qquad \qquad \frac{n_X}{T^3} \sim g_\star^{1/2} \frac{x_{FO}}{\langle \sigma |\nu| \rangle m_X M_{\text{pl}}}$$

where  $x = m_X/T$ . For weakly interacting particles  $m_X/T_{FO} = \mathcal{O}(10 - 20)$ .

This is an important and beautiful result: the relic abundance is inversely proportional to the annihilation cross-section and dark matter particle mass.



Agreement with the observed abundance  $\Omega_{dm} \sim 0.25$  requires  $\sigma \sim 1 p b a rn$ 

Which dark matter?

### Massive neutrinos?



The Cowsick-McLelland bound The Standar Model neutrinos decoupled from matter at  $T \sim 1 MeV.\,$  At this temperature, their abundance, per neutrino specie, was

$$n_\nu = \frac{3}{4} n_\gamma \propto T^3$$

(assuming no large leptonic asymmetry)

Soon after decoupling, the eletrons and positrons became NR and annihilated each others into photons. The temperature of the photons decreased more slowly than that of neutrinos so that (exercice)

$$T_{\nu} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma}$$

The bottom line is that today  $T_{\nu}=1.96K$  and

$$n_{v} = 112 cm^{-3}$$

per neutrino specie.

If the neutrinos are massive, we get that they contribute

$$\Omega_{\nu,0} = \frac{\sum_{\nu_i} m_{\nu_i}}{94 \text{ eV}}$$

which gives a bound on neutrino masses.

**Ordinary or baryonic matter** There is much more matter than antimatter around us.

If there were anti-galaxies somewhere, we would probably see spectacular flashes of  $\gamma$  rays in collisions of galaxies.

The parameter  $\eta = n_b/n_\gamma$  tells us that there are much more photons than baryons. It is also a measure of the **baryon asymmetry** of the universe.

Indeed

$$\frac{n_{\rm B}}{n_{\gamma}} = \frac{n_{\rm b} - n_{\bar{b}}}{n_{\gamma}} \sim \frac{n_{\rm b} - n_{\bar{b}}}{n_{\rm b} + n_{\bar{b}}} \bigg|_{\rm T > m_{\rm b}} \sim \left. \frac{n_{\rm b}}{n_{\gamma}} \right|_{\rm T < m_{\rm b}}$$

is a constant if the number of photon does not change and baryon number is conserved

(This is a poor approximation since photons are created in annihilation processes - we should use the entropy density instead of  $n_{\gamma}$  but the idea is the same)

Hence, the asymmetry between matter and antimatter is small  $\mathcal{O}(10^{-10})$ 

What about a baryon symmetric universe?

If there were as many baryons as antibaryons initially, on the course of the universe they would start annihilating when T  $\lesssim 1 GeV.$ 

From the dark matter lesson, we know that their would be a residual abundance of baryons (equal to antibaryons) of order

$$n_b \propto \frac{1}{\sigma}$$

where the annihilation cross-section  $\sigma \propto 1/m_{\pi}^2$ .

Calculations show that  $T_{FO} \sim 20 MeV$  and that the residual abundance of baryons would be

$$n_b/n_\gamma \sim 10^{-20}$$

This is called the "annihilation catastrophe".

Hence there must be a baryon asymmetry. Still, would be nice if we could explain why it is so small.

Sakharov (1967) has proposed a scenario called "baryogenesis".

He showed that a baryon asymmetry might arise from a baryon symmetric universe provided

- 1. Baryon number is not conserved
- 2. C and CP symmetries are violated
- 3. There is departure from thermal equilibrium
# Baryon non-conservation this condition is pretty obvious

All know processes conserve a quantum number called baryon number, eg

$$n \rightarrow p + e + \bar{\nu}$$

which has  $\Delta B = 0$ .

The lightest baryon is the proton and processes like

 $p \rightarrow \pi_0 + \bar{e}$ 

have never been observed. The current limits on this decay channel is

 $\tau_p > 1.6 \cdot 10^{33}\,\text{yr}$ 

On the other hand baryon number is only a global symmetry of the classical SM lagrangian.

It is broken at the quantum level by so-called chiral anomalies.

Moreover baryon number is not conserved in grand unification schemes, like SU(5).

# C and CP violation

Baryon number changes sign under C and CP.

A state with zero baryon number is thus an eigenstate of C and CP.

If C (CP) is conserved

$$[C(CP),H] = 0$$

then

 $\langle B \rangle(t) = 0$ 

for all time t.

C and P are maximally violated by weak interactions

while CP is almost conserved

$$\frac{\Gamma(K_L \to l^+ \nu \pi^-) - \Gamma(K_L \to l^- \bar{\nu} \pi^+)}{\Gamma(K_L \to l^+ \nu \pi^-) + \Gamma(K_L \to l^- \bar{\nu} \pi^+)} = (3.27 \pm 0.12) 10^{-3}$$

Why C violation is not sufficient



### Departure from thermal equilibrium

In thermal equilibrium

$$f_{b}(k) = \frac{1}{e^{(E_{b} - \mu_{b})/T} + 1} \quad \text{and} f_{\bar{b}}(k) = \frac{1}{e^{(E_{\bar{b}} - \mu_{\bar{b}})/T} + 1}$$
  
with  $E_{b} = \sqrt{k^{2} + m_{b}^{2}}$  and  $E_{\bar{b}} = \sqrt{k^{2} + m_{\bar{b}}^{2}}$ .

CPT symmetry gives  $m_b=m_{\bar b}$  while in chemical equilibrium  $b+\bar b\leftrightarrow \gamma+\gamma$ 

 $\mu_b = -\mu_{\overline{b}}$ 

If, moreover, processes which do not conserve B-number are in equilibrium,  $b+b\leftrightarrow\gamma+\gamma,$  then

$$\mu_b = \mu_{\bar{b}} = 0$$

Thus

$$f_{\mathfrak{b}}(k) = f_{\bar{\mathfrak{b}}}(k)$$

No asymmetry may be generated in thermodynamic equilibirum.

## A simple scenario

Out-of-equilibrium decay of a massive particle

Let

Take a pair of X and  $\bar{X}.$  Their decay produces on average a baryon asymmetry

$$B_{X} = rB_{1} + (1 - r)B_{2} - \bar{r}B_{1} - (1 - \bar{r})B_{2} = (r - \bar{r})(B_{1} - B_{2})$$

If C, CP are conserved,  $r = \overline{r}$  and there is no asymmetry. Idem if  $B_1 = B_2$  of course.

Consider a thermal bath of X and  $\overline{X}$  at temperature T.

We need the X and  $\overline{X}$  decay processes to take place out-of-thermal equilibrium.

This may happen if

 $\Gamma_X \lesssim H$ 

when  $T \sim m_X$ .

The abundance of X and  $\bar{X}$  is  $n_X = n_{\bar{X}} \sim T^3$ . Dividing by the entropy density  $s \sim g_{\star}T^3$  gives a baryon asymmetry

$$\epsilon_{\rm B} \approx \frac{(r-\bar{r})(B_1-B_2)}{g_{\star}} \sim \eta$$

To actually compute the baryon asymmetry we of course need a specific model. See for instance the lecture on leptogenesis by Pilar Hernandez.

# **Chapter Four**

# Structures formation and Inflation

#### **Structures formation**

So far we have considered a universe with perfect isotropy and homogeneity. However we see many structures in the universe on cosmological scales: galaxies, clusters of galaxies and superclusters. We may characterise inhomogeneities on a given size by their mean energy density contrast

$$\Delta = \frac{\delta\rho}{\rho}$$

We saw this figure in the first lecture



On the largest scales  $\Delta \leq 1$  and we are in a linear regime (*ie* things are simple).

The aim is to study the evolution of  $\Delta(z)$  and doing so to relate observations on the largest scales to cosmological parameters.

The universe at large z is relatively simple. As in the previous section we may say that it contains a different ideal fluids (different form of matter and radiation, possibly a cosmological constant).

The ideal fluids are inhomogeneous but we assume  $\Delta \ll 1$  . We need four basic equations to describe our fluid. They are

- 1. Eq. for conservation of energy = continuity equation
- 2. Euler equation = Newton applied to a fluid
- 3. Poisson equation = relates newtonian potential to energy density
- 4. Eq. of state = relation between pressure and energy density

In a linear approximation we get a wave-equation for  $\Delta$ 

1. Neglecting gravity and expansion but not pressure we have the usual sound wave equation.

In a mode decomposition  $\Delta(x,t)\sim \Delta_k(t)e^{ikx}$ 

 $\ddot{\Delta} + c_s^2 k^2 \Delta = 0$ 

The solutions are simply oscillatory waves

 $\Delta \sim e^{\pm i \omega t + i k x}$ 

with  $\omega = c_s k$ , where  $c_s = \sqrt{\partial p / \partial \rho_{ad}}$  is the speed of sound of the fluid.

2. Introducing gravity and pressure but keeping the background static we have one extra term in the wave equation

$$\ddot{\Delta} + \left(c_s^2 k^2 - 4\pi G\rho\right)\Delta = 0$$

with  $\rho$  the average density of the fluid.

Let  $k_J = \sqrt{4\pi G \rho/c_s^2}$ .

If  $k \gg k_J$  we have oscillatory solutions (gravity negligible) as before.

If  $k \ll k_J$  we have exponentially growing and decaying modes

 $\Delta_{\pm} \sim e^{\pm \Gamma t + i k x}$ 

with  $\Gamma\approx\sqrt{4\pi G\rho}.$ 

This is called the **Jeans instability**. It arises because gravity is attractive and tends to amplify inhomogeneities.

3. Introducing expansion brings two effects. There is an extra term

$$\ddot{\Delta} + 2H\dot{\Delta} + \left(c_s^2k^2 - 4\pi G\rho\right)\Delta = 0$$

with H the Hubble parameter. This term will damp (freeze the evolution) of modes such that  $k \lesssim H$ .

The most important effect however is that (asusming a MD universe for instance)

$$ho \propto a^{-3}$$

ie the attractive effect of the background is diluted by expansion.

For  $k \ll k_J$ , we get (in MD case) growing and decaying solutions again but

$$\Delta_+ \sim t^{2/3}$$
 and  $\Delta_- \sim t^{2/3}$ 

This is an important result

$$\Delta_+ \sim t^{2/3} \propto a(t)$$

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# Primordial inhomogeneities and the CMB

We observe  $\Delta\sim 1$  on scales  $\mathcal{O}(10Mpc).$  From our matter fluid analysis, we expect  $\Delta\sim 10^{-3}$  at  $z\sim 10^3.$ 

This redshift  $z_{LS}$  correspond to the epoch of last scattering between light and matter (aka recombination).

For  $z > z_{LS}$  we have

- Hydrogen fully ionized
- Thomson scattering  $\gamma + e^- 
  ightarrow \gamma + e^-$
- Coulomb scattering  $e^- + p \rightarrow e^- + p$

Photon-Baryon Fluid

• Photon pressure resists gravitational compression

Acoustic oscillations of photon fluid

Since  $\rho_M \propto T^3$  and  $\rho_R \propto T^4,$  we have for adiabatic perturbations (true curvature perturbations)

$$\Delta = 3\Theta \equiv \frac{3}{4} \frac{\delta \rho_{\rm R}}{\rho_{\rm R}}$$

with

$$\Theta = \frac{\delta T}{T}$$

At  $z \sim z_{LS}$ 

- Recombination into neutral hydrogen (less free electrons)
- Decoupling when  $\Gamma_{thomson} \lesssim H$  (subtle problem) gives  $z_{LZ} \sim 10^3$ , T  $\sim 3000 K \sim 0.25 eV$
- The universe becomes transparent...

Just like matter, the temperature inhomogeneities satisfy a simple wave equation (we neglect gravity)

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

I have used another time coordinate called the conformal time.

$$\eta = \int \frac{dt}{a}$$

By construction, a particle that moves a the speed of light, travels a distance  $\eta$  in a time interval  $\eta$ .





Assuming  $\dot{\Theta}(0) = 0$ :

 $\Theta(c_s\eta_{ls}) = \Theta(0)\cos(kc_s\eta_{ls})$ 

sound horizon  $= c_s \eta_{ls}$ 

Peaks in the Power Spectrum = Variance of  $\Theta(k, \eta_{ls})!$ 

$$k_n = \frac{n\pi}{c_s \eta_{ls}}$$



Evolution in time  $\Theta(\eta)$  until  $\eta_{ls}$  for  $k\ll$   $1/c_s\eta_{ls},\ k=\pi/c_s\eta_{ls}$  and  $k=2\pi/c_s\eta_{ls}$ 



The same but shows the succession of peaks in  $\Theta^2$  at  $\eta=\eta_{ls}$  in units of  $\pi/c_s\eta_{ls}$ 

The  $C'_{l}s$ 

The temperature anisotropies at last scattering are expanded in spherical harmonics

$$\Theta_{LS}(\theta, \varphi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \varphi)$$

By definition  $\langle \alpha_{lm} \rangle = 0$  but

$$\langle a_{lm}a_{l'm'}^{\star}\rangle=\delta_{ll'}\delta_{mm'}C_l$$

For high l's, we have many m's with the same variance. We expect

$$\frac{\Delta C_l}{C_l} \propto 1/\sqrt{2l+1}$$



Angular Anisotropy

$$\theta \approx \frac{\lambda}{D}$$

 $\lambda \approx c_s \eta_{ls}$   $D = comoving distance \approx \eta_0$ 



# Flat Universe:

$$\eta \propto (1+z)^{-1/2}$$
  $\frac{\eta_{ls}}{\eta_0} \approx \frac{1}{30} \approx 2^\circ$   $l_1 \approx 200$ 

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### Two elementary things we learn from these data

1. The position of the first peak is related to the size of the horizon at last scattering. Assume you know the latter. By measuring the position of the peak we measure the angular size of the horizon.



 $\theta_{H} < \theta_{H}|_{flat}$ 

 $\theta_{\rm H} = \theta_{\rm H}|_{\rm flat}$ 

 $\theta_H > \theta_H|_{flat}$ 



2. On the largest scale, we are probing the primordial inhomogeneities (not reprocessed by microphysical processes).

We know since COBE that  $\Theta \sim 10^{-5}.$ 

Remember that, from large scale structures, we expected  $\Delta=3\Theta\sim10^{-3}$  at last scattering ?

This is the best indication we have for the existence of dark matter



Now including gravity and baryons

$$m_{eff}\ddot{\Theta} + k^2 c_s^2 \Theta \approx -m_{eff} k^2 c_s^2 \Psi$$

$$m_{eff} pprox 1 + rac{3
ho_b}{4
ho_\gamma}$$

# $\Psi = Newtonian$ potential perturbation

1.  $\rho_b \ll \rho_\gamma$  and *static* potential  $\Psi$ :

$$\Theta 
ightarrow \Theta_{eff} = \Theta + \Psi$$

Sachs-Wolfe effect

With  $\Theta(0)=-\frac{2}{3}\Psi,\;\dot{\Theta}(0)=0$  (adiabatic initial conditions)

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$$\Theta_{\rm eff} = \frac{1}{3} \Psi(0) \cos(kc_{\rm s}\eta_{\rm ls})$$

- 2. Baryon drag:  $\rho_b/\rho_\gamma$  and  $\Psi$  constant
  - Baryons: greater compression in potential well
  - Redshift not affected

$$\Theta_{eff} = \frac{1}{3}\Psi(1 + 9\rho_b/4\rho_\gamma)\cos(kc_s\eta_{ls}) - \frac{3\rho_b}{4\rho_\gamma}\Psi$$

- Enhances compression peaks over rarefaction ones!
- In particular, enhances first peak!

# Summary of WMAP results

$$\begin{split} \Omega_b &= 0.047 \pm 0.006 \\ \Omega_M &= 0.29 \pm 0.07 \\ H &= 72 \pm 5 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \\ \tau_0 &= 13.4 \pm 0.3 \,\text{Gyr} \\ \Omega_0 &= 1.02 \pm 0.02 \\ \Omega_\Lambda &= 0.73 \pm 0.04 \end{split}$$

$$\frac{p_{\Lambda}}{\rho_{\Lambda}} = -1 \pm 0.2$$
  
n = 1.05 ± 0.09

# Inflation

An early phase of Accelerated Expansion (Inflation)

- $\bullet$  Solves the flatness problem  $\Omega_0\approx 1$
- Solves the horizon problem
- Predicts a scale invariant Harrison-Zeldovich spectrum of fluctuations  $n_s\approx 1$
- Predicts adiabatic fluctuations
- all

Consistent with CMB datas...

### **Flatness problem**

The CMB data tell us that the universe is flat to a very good approximation. From Friedmann

 $|1 - \Omega| = \frac{|K|}{a^2 H^2} \propto \dot{a}^{-2}$ 

while Raychaudhuri

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

For both matter and radiation  $\dot{a}$  decreases.

Take  $|1 - \Omega| = \mathcal{O}(10^{-2} \text{ today})$ .

Then at  $T_{EQ}\sim 30000 K$ 

 $|1-\Omega| = \mathcal{O}(10^{-6})$ 

while at  $T \sim 1 MeV$ 

 $|1-\Omega|=\mathcal{O}(10^{-18})$ 

Conclusion: the geometry of the universe had to be very very very very very close to flat for the universe to appear flat today



A simple remedy is to make the size of the universe very large, much larger than our horizon.

We can achieve this dynamically if the universe goes through a phase of accelerated expansion,

 $\ddot{a} > 0$ 

then

$$|1-\Omega|\propto \dot{a}^{-2}\to 0$$

ibf How much inflation do we need?

Take for instance  $p \approx -\rho$ . Then  $H \approx$  constant and

$$a = a_i e^{H(t-t_i)}$$

For,say,

$$H \sim \frac{(10^{16} GeV)^2}{M_{pl}} \sim 10^{14} GeV \qquad \text{and} \qquad \Delta t \sim 10^7 t_{pl} \sim 10^{-36} s$$

the scale factor growths by a factor of

$$a/a_i \sim e^{100}$$

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# The horizon problem solved

Our universe is very uniform, even on the largest scale we have access to

 $\sim H_0^{-1} \approx 10^{28} cm$ 

At  $T^{15}GeV,$  say, this distance was equal to  $\sim 10^{-28}H_0^{-1}\sim 1cm.$ 

This was small, but much larger than the particle horizon = causally connected region at  $T^{15}GeV,~d_{H}\sim H^{-1}\sim 10^{-14GeV}\sim 10^{-28}cm.$ 

Inflating this small distance by a factor of  $10^{28} \sim e^{65}$  would give a simple solution to the horizon = homogeneity problem.

#### How to make inflation?

The most economical, albeit ad hoc way is to take a scalar field  $\phi$  with potential

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

Suppose that the scalar field is, for some reason, presumably random chance, homogeneous, shifted away from its minimum and with small kinetic energy.

Then

$$\rho \approx \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2 \approx \frac{1}{2}m^2\varphi^2$$

and

$$p\approx \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}m^2\varphi^2\approx -\frac{1}{2}m^2\varphi^2$$

giving

 $p\approx -\rho$ 

all we need to inflate.
## What else is inflation good for?

Inflation is a source of primordial inhomogeneities.

This is a quantum effect, somewhat analogous to the phenomenon of Hawking radiation by a black hole.

For inflation, it has to do with the fact that in an inflationary background (massless or almost massless) scalar fields feel a reversed harmonic oscillator effective potential.

While  $\langle \delta \phi \rangle = 0$ , the correlator = power spectrum is non-vanishing

$$P_{\varphi}(k) = \langle (\delta \varphi)^2 \rangle \propto \frac{GH^2}{k^3}$$

For  $H \sim$  constant, the spectrum of fluctuations is scale-invariant *ie* 

$$\langle \delta \phi^2(\mathbf{x}) \rangle \propto \int d^3 k P_{\phi}(\mathbf{k}) \propto \int \frac{dk}{k} G H^2$$

there is the same power per log interval of k.

This feature, called the Harrison-Zeldovich spectrum, is supported by both the CMB and the large-scale structures surveys.

"For every complex natural phenomenon, there is a simple, elegant, compelling, wrong explanation"

Th. Gold